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Comparison of two fractal interpolation methods

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HIGHLIGHTS

- High frequency component and rough surface relies on high fractal dimension.
- The MD method's randomness is prominent, suitable for simulating aperiodic surface.
- The WM method has strong periodicity, suitable for simulating periodic surface.

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ABSTRACT

As a tool for studying complex shapes and structures in nature, fractal theory plays a critical role in revealing the organizational structure of the complex phenomenon. Numerous fractal interpolation methods have been proposed over the past few decades, but they differ substantially in the form features and statistical properties. In this study, we simulated one- and two-dimensional fractal surfaces by using the midpoint displacement method and the Weierstrass–Mandelbrot fractal function method, and observed great differences between the two methods in the statistical characteristics and autocorrelation features. From the aspect of form features, the simulations of the midpoint displacement method showed a relatively flat surface which appears to have peaks with different height as the fractal dimension increases. While the simulations of the Weierstrass–Mandelbrot fractal function method showed a rough surface which appears to have dense and highly similar peaks as the fractal dimension increases. From the aspect of statistical properties, the peak heights from the Weierstrass–Mandelbrot simulations are greater than those of the middle point displacement method with the same fractal dimension, and the variances are approximately two times larger. When the fractal dimension equals to 1.2, 1.4, 1.6, and 1.8, the skewness is positive with the midpoint displacement method and the peaks are all convex, but for the Weierstrass–Mandelbrot fractal function method the skewness is both positive and negative with values fluctuating in the vicinity of zero. The kurtosis is less than one with the midpoint displacement method, and generally less than that of the Weierstrass–Mandelbrot fractal function method. The autocorrelation analysis indicated that the simulation of the midpoint displacement method is not periodic with prominent randomness, which is suitable for simulating aperiodic surface. While the simulation of the Weierstrass–Mandelbrot fractal function method has strong periodicity, which is suitable for simulating periodic surface.

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1. Introduction

Fractal refers to the object that part of the body is similar to the whole in some way, and has been considered as a powerful tool to study complex shapes and structures in nature [1–4]. In particular, as research interest in complexity increases, the fractal theory has become a meaningful research subject to provide useful geometric characterizations of complexity [5–8]. Previous studies have revealed that the fractal with the self similarity and robust features is the organizing principle or mechanism for the generation and maintenance of large complex systems [9–12].

A lot of research has been carried out on fractal objects over the past few decades [13–16]. In 1975, Mandelbrot solidified the predecessor thought achievement and mathematical development in coining the word "fractal" and founded the fractal geometry [17,18]. Fractal theory has been developed rapidly since then, and has been widely used in many fields such as natural science, social science, cognitive science and so on [19]. Modeled fractals can take a variety of forms, including fault lines, heart sounds, geometrical optics, animal coloration patterns, digital images, soil pores, ocean waves, electrochemical patterns, circadian rhythms, etc. [20–22].

Numerous fractal interpolation methods have been proposed during the past few decades, including the fractional Brownian motion simulation method, the inverse Fourier transformation method, the Weierstrass–Mandelbrot fractal function simulation method, the fractal interpolation function method, and the time series simulation method [23–27]. The fractional Brownian motion simulation method can be further classified into the midpoint-displacement method, the Poisson Faulting method and Successive random additions method [28–30]. In particular, the midpoint displacement (MD) method and the Weierstrass–Mandelbrot (WM) fractal function method are the most commonly used fractal methods [31–33] by researchers. These fractal methods have often shown substantial differences in their form features and statistical properties [34,35]. For example, a previous study has assessed three fractal surface modeling methods in their synthesizing speed, accuracy and surface root-mean-square deviation [36]. The study showed that, although the synthesizing speed of discrete Fourier transform method is slightly slower than that of random midpoint displacement, its synthesizing accuracy and root-mean-square deviation controllability are better than those of the random midpoint displacement and two-dimensional Weierstrass–Mandelbrot function.

The objective of this study is to assess two fractal surface modeling methods in one and two dimensions, and particularly, to (1) examine the form features of the one- and two-dimensional surface simulations with the midpoint displacement method and the Weierstrass–Mandelbrot fractal function method; (2) analyze and compare their statistical characteristics with different fractal dimensions; and (3) make the autocorrelation analysis of the two fractal methods for their periodic variation regulations.

2. Methodology

2.1. The midpoint displacement (MD) method

The MD method is a classical and direct application of fractal Brownian motion, and has been widely used to generate natural scene images with complex shape, such as mountain elevation map and cloud image [37–39]. As the algorithm is easy to implement and runs fast, it is one of the frequently used random fractal algorithms [40]. The basic principle is the variance power law relation based on ΔX, which can be expressed as follows:

$$E[X(t+\Delta t) - X(t)^2] = |\Delta t|^{2H} \sigma^2 \tag{1}$$

where σ is the mean square height; the conversion relationship between Hearst exponent H and fractal dimension D is defined as $D = 2 - H$; $X(t)$ is a random variable which follows a normal distribution.

The MD method is the displacement of elevation at the midpoint of a line segment, then identifies the midpoint of the partitioned line segments, and displaces again. This process is carried out recursively until a certain spatial resolution is satisfied. The point displacement can be expressed as follows:

$$X_n\left(\frac{t_1 - t_2}{2}\right) = \frac{1}{2}(X_{n-1}(t_1) + X_{n-1}(t_2)) + \varDelta_n \tag{2}$$

where $\varDelta_n$ is a Gaussian random variable with the mean 0 and the variance $\varDelta_n^2$; it is deduced by n steps, and the $\varDelta_n^2$ can be expressed as follows:

$$\varDelta_n^2 = \frac{\sigma^2}{(2^n)^{2H}}(1 - 2^{2H-2}). \tag{3}$$

It has a mesh construction mode when the two-dimensional surface is simulated with the MD method. In general, the commonly used mesh construction methods include the triangular mesh, quadrilateral mesh, diamond mesh, hexagonal mesh, and parameter quadrate mesh [41–43]. For purposes of comparison, this study uses the quadrilateral mesh as the mesh construction mode.