Physica A 469 (2017) 777–788

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Cascading failures in interdependent networks due to insufficient received support capability

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HIGHLIGHTS

- A capability based dependency model is proposed, which allows us to discuss the redundancy degree and the number of support-dependence links separately.
- The interdependent network without redundant support-dependence links is extremely vulnerable.
- Increasing the amounts of support-dependence links can enhance the robustness of interdependent network without changing its redundancy degree.
- Improving the redundancy degree can also make the interdependent network more robust.

ARTICLE INFO

Article history:
Received 9 April 2016
Received in revised form 22 October 2016
Available online 17 November 2016

Keywords:
Robustness
Interdependent networks
Support capability
Required capability
Redundancy degree

ABSTRACT

We propose a capability based dependency model of interdependent network that takes two node dependency properties into account. One is support capability and the other is required capability. The redundancy degree of an interdependent network is also defined, whose value is the ratio of its total support capability and total required capability. Through the numerical simulations, we found that: (1) Interdependent networks without redundant support-dependence links are extremely vulnerable, even the failure of one node could cause the collapse of whole network; (2) Increasing support-dependence links and redistributing the nodes' dependency properties can enhance the robustness of network without changing its redundancy degree; (3) Improving the redundancy degree could enhance network robustness without adding support-dependence links. These conclusions enlighten the design of interdependent networks: when network's redundancy degree is fixed, we can take strategy from results (2), and when network structure is settled, we can apply strategy from results (3).

1. Introduction

The critical infrastructure (CI), such as the transportation networks, power grid and water supply networks, is one of the most essential components for the functioning of society and economy. In research, CI is often treated as a complex network since it consists of a mass of components. The robustness of a CI network is the most important index to measure its ability to resist changes, and it is generically defined as the survivability when a CI network meets failures caused by inside errors, outside attacks, or any other disasters.

There has been much research on the robustness of individual network [1–3] in the past decades. However, with the development of information technology, CI tends to be accessed by the Internet and forms the so called Cyber-Physical

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http://dx.doi.org/10.1016/j.physa.2016.11.068
0378-4371/

Systems (CPS). In CPS, the *physical* region and the *cyber* region are strongly connected. Furthermore, different CIs are coupled together [4,5]. This means that CIs tend to be interdependent networks, so the individual network models can no longer apply to CI networks.

In 2010, Buldyrev et al. [6] proposed a theory about the cascading failure of interdependent networks. In this theory, each node depends on and supports no more than one node in the other network, so it is called one-to-one dependency model. Since then, a lot of research on cascading failures of interdependent networks has been done. Some of them focus on the coupling features of interdependent networks, such as coupling strength [7], coupling with different types of networks [8], coupling with different size networks [9] and coupling preference [10,11]. While there is also much work focusing on the network structural properties, including clustering [12–14], degree correlation [15–17] and directed and weighted networks [18]. Meanwhile, random breakdown [19] and targeted attack [20,21] are studied to design resilient networks under different threats.

In the real world, when a node fails, it does not necessarily mean the node becomes orphan, either caused by disconnection from giant component or losing support nodes. It could also be caused by *overload*: the load of a node exceeds its capacity. This loaded-induced cascading failure is very important in power grids, energy supply systems and transportation networks [22], and it draws extensive attention [23–26].

Another model, multiple dependency model, is introduced by Shao [27,28]. In this model, constraint of support-dependence relations is loosened—each node can have more than one support nodes in the other network, and a node can function when at least one of its support node survives. Not only is the robustness of two coupled networks studied, the robustness of multiplex networks [29–32] and network of networks has been researched [33,34].

The research mentioned above pays less attention to the properties of the support-dependence relations, especially the physical meaning of support-dependence relations. We wonder that, in one-to-one dependency model and multiple dependency model, why a node can survive only if it has at least one support node in the other network, and whether the following condition exists: A node should have more than a certain amount of support nodes to survive. We solve the problems by introducing a capability based dependency model, and the concepts of *support capability* and *required capability* are presented to explain the physical meaning of the support-dependence relations. In our model, a node can function only if it has received sufficient support capability, in other words, a node may fail due to insufficient *received support capability*. For example, the smart grid consists of power station network and communication station network, and the two networks are coupled together. The smart grid encourages distributed generation, so the electric power that maintains a communication station functioning may come from different power stations. To keep a communication station functioning normally, there must be plenty of support power stations (e.g. at least half of all support power stations, less than a half may make the voltage lower than the rating voltage) survival. On the other hand, one power station may receive control commands from different communication stations, and different communication stations may provide different control commands. Only being supported by sufficient functioning communication stations can make the power station continue functioning. So for simplicity, insufficient support capability from power stations will make the communication station failed and vice versa.

We build our model drawing lessons from the secret sharing problem [35,36], which can be described as: A secret is divided into m pieces and can be decrypted by more than any k $(k < m)$ pieces, but we cannot get any information about the secret if we get fewer than k pieces of the secret. In our model, a node is initially supported by m nodes, and only being supported by more than any k $(k < m)$ nodes can keep it functioning. Being supported by fewer than k nodes would make the node failed.

Our model is more universal to describe the interdependent networks in the real world. The multiple dependency model is just a special case in our model. The concept of *redundancy degree* of interdependent network is defined, which represents the ratio of the total support capability of all nodes to the total required capability of all nodes. Numerical simulations show that the interdependent network is quite vulnerable when there are no redundant support-dependence links, and if there are no autonomous nodes, the network will collapse soon. We could enhance the robustness of interdependent networks by improving the redundancy degree. Besides, it is also possible to make a more robust interdependent network by adding support-dependence links without changing its redundancy degree.

This paper is organized as follows. We first introduce our capability based dependency model in Section 2. Then we perform the numerical simulations and present the simulation results in Section 3. At the end of paper, we make conclusions and discussions.

2. The model

In our model, the interdependent network whose size is N consists of two interdependent sub-networks called network A and network B, and the two networks have the same size. $N^A = N/2$ and $P^A(k)$ represent the size and degree distribution of network A, analogously, $N^B = N/2$ and $P^B(k)$ represent the size and degree distribution of network B. We call the links between the two subnetworks as *support-dependence links*, which can deliver capability from the source node to the target node. One thing to note is that the terms 'support' and 'dependence' do not have any meanings of *AND-operation* or *OR-operation*. In initial stage (stage 0), there are $c_0^{BA}N^A$ support-dependence links from nodes in network B to nodes in network A and $c_0^{AB}N^B$ support-dependence links from nodes in network A to nodes in network B. Each support-dependence link delivers equal capability, denoted by l. The support-dependence links are distributed randomly. We take the adding support-dependence links from network A to network B for example, the adding process is as follows: Each time we randomly choose