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Q1 # Stochastic sensitivity analysis of noise-induced order-chaos transitions in discrete-time systems with tangent and crisis bifurcations

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HIGHLIGHTS

- Noise-induced order-chaos transitions are studied.
- Stochastic discrete-time systems with tangent and crisis bifurcations are considered.
- Stochastic sensitivity function technique is used for parametric analysis.

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ABSTRACT

We study noise-induced order-chaos transitions in discrete-time systems with tangent and crisis bifurcations. To study these transitions parametrically, we suggest a generalized mathematical technique using stochastic sensitivity functions and confidence domains for randomly forced equilibria, cycles, and chaotic attractors. This technique is demonstrated in detail for the simple one-dimensional stochastic system, in which points of crisis and tangent bifurcations are borders of the order window lying between two chaotic parametric zones. A stochastic phenomenon of the extension and shift of this window towards crisis bifurcation point, under increasing noise, is presented and analyzed. Shifts of borders of this order window are found as functions of the noise intensity. By our analytical approach based on stochastic sensitivity functions, we construct a parametric diagram of chaotic and regular regimes for the stochastically forced system.

1. Introduction

A relationship between order and chaos is one of the challenging problems of the modern nonlinear dynamics. An elucidation of the underlying mechanisms of transitions from order to chaos and back, attracts attention of many researchers [1–4]. In deterministic systems, various scenarios of transitions to chaos are known, namely through the period-doubling bifurcations [5,6], quasiperiodicity [7,8], intermittency [9,10], crisis bifurcation [11,12], and so on (see, for instance [1,13]).

In stochastic systems, a weak noise can cause a transition to chaos even if the initial unforced deterministic system is regular. This phenomenon of noise-induced chaos was studied in Refs. [14,15]. In some stochastic systems, an inverse noise-induced transition from chaos to order can be observed [16–18]. A phenomenon of noise-induced ordering is actively investigated within the context of the stochastic resonance [19–23].

A standard criterion of the transition of the system from regular to chaotic regime is a change of the sign of largest Lyapunov exponent from minus to plus. This criterion only indicates the transition from order to chaos, but does not bring

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to light the underlying probabilistic mechanisms of order-chaos transitions connected with the interplay of the stochasticity and nonlinearity.

Because of nonlinearity, phase portraits of deterministic systems can be highly non-homogeneous. This is an underlying reason of various noise-induced effects. For example, a deterministic system can possess coexisting attractors, both regular and chaotic, with basins of attraction divided by separatrices. So, dynamics of such multistable systems can be regular or chaotic depending on the initial state. Under stochastic disturbances, random trajectories overcome these separatrices, and as a result, various stochastic phenomena occur [24–26], including noise-induced order-chaos transitions [27,28]. Note that the noise-induced transition of system dynamics from regular to chaotic can occur also in monostable but excitable systems [29–31]. The effect of noise is significantly enhanced near the bifurcation points.

In present paper, we focus on the study of the influence of noise on tangent and crisis bifurcations in nonlinear discrete systems.

In deterministic systems, the tangent bifurcation generates a transition to chaos via type-I intermittency. An analysis of the noise-induced intermittency attracts attention of many researchers [1,32–36]. An influence of random disturbances on the deterministic system with crisis bifurcations was studied in Refs. [37–39] numerically. To analyze these noise-induced phenomena parametrically, one has to take into account both stochastic sensitivity of attractors and geometry of their basins of attraction.

As well known, an analysis of the stochastic systems is based on the investigation of probabilistic distributions of their random states. A detailed description of these distributions is given by the Kolmogorov–Fokker–Planck equation for continuous-time systems, and by the Frobenius–Perron integral equation for discrete-time systems. To avoid the complexity of a direct solution of these equations, various asymptotics and approximations can be used [40,41]. One of them is a stochastic sensitivity function (SSF) technique. This technique has been used for the constructive description of probabilistic distributions for both continuous [42] and discrete-time [43] systems.

In present paper, we suggest the unificated approach to the parametric analysis of the noise-induced transitions between order and chaos. Our approach is based on the generalized SSF technique covering randomly forced equilibria, cycles, and chaotic attractors of discrete-time systems. A brief theoretical background of this approach is given in the Appendix for the one-dimensional case.

In our study, we use a simple but representative one-dimensional nonlinear discrete system with the unimodal map. Under the variation of the control parameter, this deterministic system undergoes tangent and crisis bifurcations, lying nearby.

In Section 2, we present this initial deterministic one-dimensional model and discuss its behavior in parametric zones separated by crisis and tangent bifurcations. In bistability zone, this system possesses a stable equilibrium and chaotic attractor. The monostability zone consists of two parts. In the first part, the system has a stable equilibrium, and in the second part, there is a single chaotic attractor. The points of crisis and tangent bifurcations are borders of the window of order, and separate this window from adjoining chaotic parametric zones. Due to such variety of deterministic dynamics, the corresponding stochastic system demonstrates diverse scenarios of order-chaos transitions. A probabilistic analysis of these scenarios is given in Section 3.

In Section 3.1, we study stochastic dynamics in the bistability zone where the stable equilibrium and chaotic attractor are separated by the unstable equilibrium. Under stochastic disturbances, random trajectories can transit from the chaotic attractor to the basin of attraction of the stable equilibrium. Such transitions are accompanied by the transition from chaos to order. For the parametric analysis of this transition, we use SSF technique and confidence domains method. Here, a novelty is to estimate the stochastic sensitivity of the borders of the chaotic attractor. To find these estimations, we use a 3-cycle of some appropriate modeling system. In this bistability zone, inverse transitions from the equilibrium to chaotic attractor can be observed. Such transitions are studied also by our general SSF technique presented in the Appendix.

In Section 3.2, we study stochastic dynamics in the monostability zone with a single stable equilibrium. Under the noise, random trajectories can jump over the unstable equilibrium, and fall within a zone of large-amplitude stochastic oscillations. After these quite long oscillations, the system returns again to the vicinity of the stable equilibrium, and so on. Such noise-induced intermittency can transform the system from order to chaos. An analysis of this noise-induced chaos we also derive on the basis of the stochastic sensitivity function technique and method of confidence domains. This analysis involves the calculation of stochastic sensitivity of the stable equilibrium and study of the mutual arrangement of the confidence intervals and unstable equilibrium.

Results of our study of noise-induced order-chaos transitions in this model are gathered in the parametric diagram.

2. Deterministic model

Consider a discrete-time nonlinear dynamic system [44]

$$x_{t+1} = f(x_t, \mu), \qquad f(x, \mu) = \mu x(1-x)(lx^2 + px + q), \tag{1}$$

where

$$l = \frac{1}{1 - s_1 + s_2 - s_3}, \qquad p = l(1 - s_1), \qquad q = l(1 - s_1 + s_2),$$
$$s_1 = r_1 + r_2 + r_3, \qquad s_2 = r_1 r_2 + r_2 r_3 + r_3 r_1, \qquad s_3 = r_1 r_2 r_3.$$

For any μ, system (1) has a trivial equilibrium $\bar{x}_0 = 0$.

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