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Transfer mutual information: A new method for measuring information transfer to the interactions of time series

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HIGHLIGHTS

- A new transfer mutual information method is proposed.
- It measures the influence of a third variable on the interactions between two variables.
- Several generalizations of the method are proposed.
- The simulations verify the utility of the transfer mutual information.

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ABSTRACT

In this paper, we propose a new method to measure the influence of a third variable on the interactions of two variables. The method called transfer mutual information (TMI) is defined by the difference between the mutual information and the partial mutual information. It is established on the assumption that if the presence or the absence of one variable does make change to the interactions of another two variables, then quantifying this change is supposed to be the influence from this variable to those two variables. Moreover, a normalized TMI and other derivatives of the TMI are introduced as well. The empirical analysis including the simulations as well as real-world applications is investigated to examine this measure and to reveal more information among variables.

1. Introduction

Many real-world systems output variables that may interact with each other. Clarifying the relationship between each two variables can be devoted to better understand the intrinsic mechanism of the systems as well as to predict the forthcoming dynamics. It is a ubiquitous issue and has long been investigated on how to quantify the relationship between two variables (or time series) [1–5]. More than one century ago, the Pearson's correlation coefficient ρ was introduced to analyze the correlations between two variables (here represented by X and Y) [6]. The Pearson's correlation coefficient works well merely in the presence of linear correlations while often fails if nonlinear correlations occur. Another problem is that a third variable may take effects on the correlations between the first two variables. In such a case, the partial correlation was introduced to measure the linear correlations between variables X and Y, in the absence of a third variable Z [7], which has been widely used to determine the order p of the autoregression (AR) model. In the partial correlation analysis, X and Y are projected on the perpendicular space of Z, into X' and Y' respectively, then the Pearson's correlation

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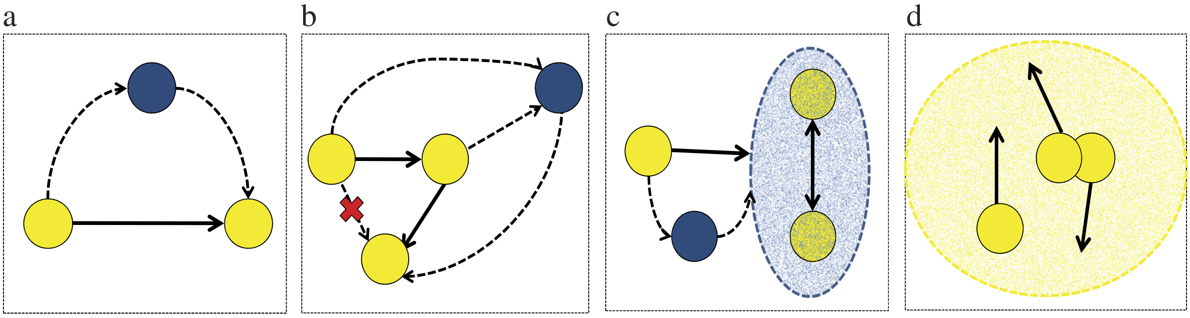

Fig. 1. A diagram for 4 possible types of relationships among variables: (a) The information flow transfers from a variable to another variable directly, or through a third variable indirectly. In such a case, the partial correlation and the partial transfer entropy [25] can measure the net information flow in the absence of the third variable. (b) Only indirect information flow while no direct one exists. (c) The influence of a third variable on the interactions between another two variables, which can be quantified by such as the relative effect (i.e. the difference between the correlation and the partial correlation) indicated in Ref. [26], as well as the TMI introduced in this paper. (d) The information contributions of the individuals to the whole system have been studied in our previous research [27].

coefficient is estimated between two projections. Equivalently, X' and Y' are estimated through the ordinary least square, i.e. resulting from the linear regression of X with Z and of Y with Z, respectively. The proposal of Pearson's correlation coefficient puts forward a huge success, get itself into one of the most classic measures in statistics, and further becomes a cornerstone of its follow-up developments in multivariate analysis like the principal component analysis [8], singular value decomposition [9], random matrix theory [10], etc. Nevertheless, with contrast to the Pearson's correlation coefficient and the partial correlation owning their limitation to further analyze nonlinear relationships [11], the nonlinear interactions between variables generally conform with the regulations of real-world systems [12–14], and in most cases they cannot be simplified as linear systems. To solve it, a candidate of the Pearson's correlation coefficient is the mutual information (MI) [15]. Following the Shannon entropy proposed by C. E. Shannon [16], the MI was introduced to quantify the linear and also the nonlinear connections between two variables:

$$I(X, Y) = H(X) - H(X|Y), \tag{1}$$

where $H(X)$ is the Shannon entropy of X: $H(X) = -\sum_x p_x \log p_x$, and p_x are the probabilities of X. $H(X|Y)$ is the conditional entropy of X in the presence of Y: $H(X|Y) = -\sum_{xy} p_{xy} \log p_{x|y}$, where p_{xy} are the joint probabilities of (X, Y), and $p(x|y)$ are the conditional probabilities of X in the presence of Y. $H(X)$ represents the uncertainty of X; $H(X|Y)$ represents the uncertainty of X in the presence of Y. Therefore, the MI is quite straightforward. It indicates the change of the information of X when another variable Y arises, and naturally, the MI is supposed to be the influence of Y on X. Moreover, $I(X, Y)$ is symmetric, i.e. $I(X, Y) = H(Y) - H(Y|X)$. Hence, $I(X, Y)$ also describes the influence of X on Y. This influence either from X to Y or from Y to X is thus considered as the mutual information transferred between X and Y. Interestingly, many more measures refer to this idea [17–19], which can be summarized as follows:

If the presence or the absence of one variable Y does make change to another variable X, then this change, if it can be quantitatively measured, is supposed to be the influence of Y to X.

The above idea is well-known and widely-accepted. The change can be estimated in diverse ways, and many methods have been developed to quantify this change. Take the transfer entropy for example [19]:

$$TE_{Y\to X} = H[X_t|X^{(t-1)}] - H[X_t|X^{(t-1)}, Y^{(t-1)}], \tag{2}$$

where $TE_{Y\to X}$ means the transfer entropy from one variable Y to another variable X, which is supposed to be the influence from Y to X. On the right side, $H[X_t|X^{(t-1)}]$ denotes the uncertainty of X at time t when we already obtain the past information of X before t, where $X^{(t-1)}$ denotes a vector of X before time t and the uncertainty is quantified by the Shannon entropy. If we introduce another variable Y, the uncertainty of X may decrease due to further acquiring the past information of Y. The change of uncertainty in X, given by the transfer entropy, is therefore considered as the influence from Y to X. The transfer entropy has been confirmed adequately for determining the magnitude as well as the direction of information flow between two coupled variables in many real-world applications [20–24], which show advantages over the traditional time-delay mutual information. More recently, a partial transfer entropy [25] was proposed to quantify the indirect influence that variables have on one another, which is similar to the partial correlation.

The relationship discussed above is confined to analyze the influence from one variable to another one, or the information flow between two variables in the absence of a third variable. For detailed information, please see Fig. 1, where a diagram for several possible types of relationships among variables are presented. Unfortunately, a crucial relationship is often unnoticed, which is also is few studied, i.e. the influence of a given variable Z on the interactions of another two variables X and Y. How to quantify this influence is a rather important issue both for natural and social concerns, which would be worth of close attention. For example, how to evaluate the influence of U.S. on the relationship between China and Russia, how to quantify the influence of a wife on the relationship between her husband and her mother, and how to determine the