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Resonance in an ensemble of excitable reaction–diffusion systems under spatially periodic force

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ABSTRACT

In this paper, we investigate the resonance collective behavior in an ensemble of excitable reaction–diffusion systems subjected to the weak signal and spatially periodic force. It is demonstrated that the resonance behavior is optimized by intermediate values of the spatial force's amplitude and frequency, which is termed spatially periodic-force-induced resonance. Moreover, we study that how the diffusion coefficient and modulation period influence the response of the system to the external weak signal, and present the mechanism of this resonance phenomenon. These findings show that spatially periodic force as intrinsic diversity might have a constructive role and shed light on our understanding of the collective behaviors of nonlinear systems driven by spatially periodic force in response to the weak signal.

1. Introduction

Stochastic resonance is a noise-induced effect demonstrating the phenomenon of signal amplification, which has drawn much attention in nonlinear science for more than 30 years [1–3]. The basic mechanism is such that the nonlinear systems' response to the external weak signal can be amplified under an appropriate strength of noise. Further noise-induced and analogous resonance phenomena have been extensively studied in diverse research areas including physics, chemistry, biological systems, and technological sciences [4–16]. The rich dynamic behaviors of these processes induced by noise or the other driving sources include coherence resonance [5], array-enhanced coherence resonance [4], diversity-induced resonance [8], vibrational resonance [6,7], just to name a few.

Recently, the influence of spatially periodic force on dynamic behaviors has been explored in the reaction–diffusion systems [17–22]. For instance, Dolnik et al. studied the role of the amplitude and wavelength of spatial periodic forcing on the hexagonal pattern of Turing structures, and they found that periodic spatial forcing interacts with the Turing structures and modifies the pattern symmetry and wavelength [17]. Page et al. investigated the effects of spatially varying parameters on pattern formation using the Gierer–Meinhardt reaction–diffusion model, and they showed that spatially heterogeneous parameters can both increase the range and complexity of possible patterns and enhance the robustness of pattern selection [18].

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Inspired by the above studies, here we are interested in the problem that how the spatially periodic force induces resonant behavior in the excitable reaction–diffusion system. Based on the methods we developed in the bistable reaction–diffusion system in Ref. [22], we explore how the spatially periodic force influences the response of the excitable reaction–diffusion system to the weak signal in this work. To the best of our knowledge, the resonance behavior in excitable reaction–diffusion system has not yet been carefully studied, although it certainly could help to better understand the collective behaviors of nonlinear systems. The paper is organized as follows. In Section 2, the model and some resonance factors are presented. In Section 3, all numerical results of resonance behaviors in excitable reaction–diffusion system relying on our observations are given in details. Finally, the last section is devoted to a brief discussion and conclusion.

2. Model

First, let us start from the simple but classical FHN model, consisting of two ordinary differential equations,

$$\varepsilon\frac{du}{dt} = u - u^3/3 - v,$$
$$\frac{dv}{dt} = u + a, \tag{1}$$

where ε is taken to be $\ll 1$ allowing one to separate the variables u and v for the fast and slow reactions, respectively. Another system parameter a characterizes the system dynamics: when $|a| < 1$, the system is in the oscillatory regime, while for $|a| > 1$ it is in the excitable one, namely, only a sufficiently large perturbation can make the system deviate from the fixed point and exhibit a large near-limit-cycle excursion.

We now turn our attention to the following excitable reaction–diffusion system by introducing inhomogeneity and spatially coupling:

$$\varepsilon\frac{\partial u}{\partial t} = u - u^3/3 - v + D\frac{\partial^2 u}{\partial x^2},$$
$$\frac{\partial v}{\partial t} = u + a + c\sin\left(2\pi f\frac{x}{L}\right) + A\sin(\omega t), \tag{2}$$

where $u(x, t)$ and $v(x, t)$ represent the action potential and potassium gradient at position $x(0 \le x \le L)$ for time t. D is the diffusion coefficient, $c\sin(2\pi f\frac{x}{L})$ is a spatial force with the spatial frequency f and amplitude c. The system is also subjected to an external weak periodic signal of intensity A and frequency ω. All our numerical results are obtained through the standard Euler approach of the reaction–diffusion equation [Eq. (2)] with periodic boundary conditions for the fixed time step $\Delta t = 0.001$ and grid distance $\Delta x = 0.1$. All the simulations will be performed with the system variables ($u(x, t)$ and $v(x, t)$) chosen from a uniform distribution in $[-0.1, 0.1]$.

To quantify the response of the system to the weak periodic signal, we define a temporal resonance factor Q_t as

$$Q_t = \sqrt{Q_{t1}^2 + Q_{t2}^2},$$
$$Q_{t1} = \frac{1}{T}\int_{T_0}^{T_0+T} X(t)\cos(\omega t)\,dt,$$
$$Q_{t2} = \frac{1}{T}\int_{T_0}^{T_0+T} X(t)\sin(\omega t)\,dt, \tag{3}$$

where $X(t) = \frac{1}{L}\int_0^L u(x, t)\,dx$, characterizing the signal output of the mean field $X(t)$ at the input frequency ω. Both sufficiently large T_0 and T are needed here, large T_0 is used to discard the transient period and large T for a proper measurement of average over time. Q_t can provide a precise measure of the response ability of the system to the weak periodic signal at fixed driver frequency ω. Furthermore, we introduce a spatial resonance factor Q_s to characterize the response of the system to the spatially periodic force at spatial frequency f,

$$Q_s = \sqrt{Q_{s1}^2 + Q_{s2}^2},$$
$$Q_{s1} = \frac{1}{L}\int_0^L Y(x)\cos\left(2\pi f\frac{x}{L}\right)\,dx,$$
$$Q_{s2} = \frac{1}{L}\int_0^L Y(x)\sin\left(2\pi f\frac{x}{L}\right)\,dx, \tag{4}$$

where $Y(x) = \frac{1}{T}\int_{T_0}^{T_0+T} u(x, t)\,dt$.