PHYSA: 17507 Model 3Gsc pp. 1–9 (col. fig: NIL)
ARTICLE IN PRESS
Physica A xx (xxxx) xxx–xxx

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Q1 # Optimization of spatial complex networks

Q2 S. Guillier [a,*], V. Muñoz [a], J. Rogan [a,b], R. Zarama [c,d], J.A. Valdivia [a,d]

[a] Departamento de Física, Facultad de Ciencias, Universidad de Chile, Santiago, Chile
[b] Centro para el Desarrollo de la Nanociencia y la Nanotecnología, Cedenna, Santiago, Chile
[c] Departamento de Ingeniería Industrial, Universidad de los Andes, Bogotá, Colombia
[d] CEIBA complejidad, Bogotá, Colombia

HIGHLIGHTS

- Growth model for spatial network subject to optimization criterion.
- Scaling exponent depends on optimization criterion.
- Transition from exponential to scale-free behavior studied.
- Discussion of network size effects.

ARTICLE INFO

Article history:
Received 25 October 2013
Received in revised form 27 May 2016
Available online xxxx

Keywords:
Spatial networks
Traffic networks

ABSTRACT

First, we estimate the connectivity properties of a predefined (fixed node locations) spatial network which optimizes a connectivity functional that balances construction and transportation costs. In this case we obtain a Gaussian distribution for the connectivity. However, when we consider these spatial networks in a growing process, we obtain a power law distribution for the connectivity. If the transportation costs in the functional involve the shortest geometrical path, we obtain a scaling exponent $\gamma = 2.5$. However, if the transportation costs in the functional involve just the shortest path, we obtain $\gamma = 2.2$. Both cases may be useful to analyze in some real networks.

1. Introduction

Q3

Complex networks have received substantial attention in recent years, because they provide a useful representation for many technological, biological and social systems [1–4]. Many of the networks in nature have common features, such as the connectivity distribution $P(k)$, which often results to behave as a power law $P(k) \sim k^{-\gamma}$ for large degree k [5–14]. Several authors have developed network models that seek to replicate these distributions. One of the most emblematic cases was proposed by Barabási–Albert [1,15,16], in which a weighted random growth model is used to generate a power law distribution with $\gamma = 3.0$. In this model in each step a new vertex appears and it is connected randomly with a vertex of the network with a probability proportional to its connectivity degree. Empirical networks show similar characteristic exponent γ, such as citation networks [5] and electronics circuits [6]. However there are other important sets of systems which exhibit a network structure with a different exponent, for example: telephone calls [7,8], World Wide Web [9], metabolic [10],

* Corresponding author.
E-mail address: sguillier@ug.uchile.com (S. Guillier).

http://dx.doi.org/10.1016/j.physa.2016.09.011
0378-4371/

movie actors [11], protein interaction [12], the internet [13] and word co-occurrence [14]. In these cases, the characteristic exponent γ ranges from 2.0 to 2.7.

Although there are variations to the Barabási–Albert model which generate complex networks with exponents different than 3.0 (*e.g.* mixing of random and preferential attachment, additional attractiveness of nodes, or introducing aging of nodes [1,17]); various authors have proposed models that yield such exponents by considering that the network is subject to a certain optimization process, with respect to a parameter relevant to the network of interest. For example, such models have been considered to describe brain functional networks where the parameter to be optimized is the coherence of coupled oscillators, representing brain regions [18,19]. Discussion of optimized networks leading to scale-free distribution has been discussed, for instance, in Ref. [20], where trees in software architecture graphs are studied, and optimization is underlying the creation of the graphs by means of the design principles involved in the software creation.

In particular, the role of optimization has been extensively studied in networks where some notion of distance between nodes is relevant. Distance between vertices through the network can be defined as the number of edges necessary to go from one vertex to another. This is important, since it allows to define a distance for arbitrary systems, such as social networks or the World Wide Web, where a physical distance is not necessarily meaningful. However, there is another equally important set of complex networks where vertices do have a definite position in space [21]. We will call them spatial networks, and they are usually related to transportation: city traffic networks, power networks, telephone wiring networks, internet, etc. In all of them, there is some kind of load being carried between vertices, which can be cars in a city traffic network or electricity in power networks. In these spatial networks, the nth vertex is placed at a fixed position (x_n, y_n) in space, and a geometric distance between vertices can be calculated, *e.g.* the Euclidean distance.

Naturally, both distances are in principle completely unrelated: two vertices directly connected by the network could be separated by a very large geometric distance. This leads to an interesting conflict when the evolution of the network is considered: if a new city appears on the map, then what new roads should be built to connect it with the pre-existent cities? The decision could be to connect it with the nearest city, or with the most important one. And, in turn, if the choice is to prioritize connections to a given city, is it better to connect them directly, thus reducing construction costs, or indirectly, passing through some other cities, in order to increase trading between them? It is therefore evident that, in a real evolving transportation network, there arises the issue of how both kinds of distances, geometrical or topological, compete.

Using these spatial networks and these distances, several authors have proposed network models based on some optimization process [22–25]. Regarding transport networks in particular, it seems natural to define two kinds of costs to optimize: the cost of constructing the network and the cost of transporting through it. If someone wants to fabricate an internet network, the cost should be related to the total length of cable that is going to be used, while if somebody wants to elaborate a railway network, the cost should be related to the total length of railroads to be built. Inspired by these examples, we will assume that the cost of constructing a network is proportional to the sum of the length of the connections. On the other hand, the transportation cost is the cost of actually moving the load through the network. For instance, for the railway network, the cost is related to issues such as fuel consumption, wear on the train wheels, etc. Notice that these transportation costs are also related to the length of the connections, but they are not equivalent to construction costs. For instance, it is possible that for a given spatial distribution of cities, building a road between cities A and B would be very expensive, suggesting that it might be better not to spend money building it; however, that road may turn out to improve connectivity in the network as a whole, so that transportation costs end up being attractive. Thus, when assessing the overall cost of a network, the cost of having to build the roads once, and then using them to move load between all the cities, must be considered as two separate contributions to the total cost. In other words, a transport network could be the result of the minimization of some cost function that on the one hand considers the cost of construction, and on the other hand considers the transportation cost. This way of thinking in a transport network has already been developed by some publications at the beginning of the 70s [26,27], and lately it has been widely studied by several authors [28–31].

On this basis, studies of various optimization criteria have been made to determine, for example, how the statistical properties of the resulting complex network acquire small-world or scale-free behavior, and, in the latter case, a range of exponents have been shown to arise. In the case of static networks, where the number of nodes is fixed, optimization can be achieved by rewiring the network, as studied in Refs. [23,32], showing that optimization can lead to small-world and scale-free networks.

Growing networks have also been studied [33–35]. For instance, in computing related networks, such as Ref. [34] where the case of Internet topology was considered, the growth is determined by competition between the cost of connecting a new node and the cost of transmission delays. Optimization of spatial growing networks has also been studied in Ref. [35], where the interplay between the timescales for assembly and for optimization of the network is discussed. Thus, in this case the optimization process involves rewiring of the network after each new node is added to the network, a rewiring which can be either global (new links can involve nodes separated by arbitrary distances) or local (new links are chosen only among nearby nodes). A variety of power-law exponents are shown to emerge.

In this work, we intend to study the effect of different optimization criteria on the degree distribution of spatial networks. In Section 2 a Global Optimization Model is discussed, where all vertices are initially known, and edges are created according to a certain optimization criterion. Then, Sections 3 and 4 deal with a Weighted Growth Optimization Model, where at each step a new vertex appears and it is then connected with a vertex of the network, again following an optimization criterion. In Section 5, a more detailed analysis of the dependence of the resulting networks on λ, along with a discussion of size effects, is carried out. Finally, in Section 6 results are summarized and discussed.

Please cite this article in press as: S. Guillier, et al., Optimization of spatial complex networks, Physica A (2016), http://dx.doi.org/10.1016/j.physa.2016.09.011