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Towards improving the MITC6 triangular shell element

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Abstract

Effective triangular shell elements are of utmost interest in engineering practice, and the MITC6a element – a 6 node quadratic general shell element of the MITC family – has been shown to significantly reduce the locking phenomena arising in bending dominated behaviours. However, for some specific combinations of midsurface geometry and boundary conditions, the MITC6a element features some non-physical displacement modes with vanishing membrane strain energy. This phenomenon is thoroughly analyzed, and a remedy based on a stabilized bilinear form is proposed. Detailed numerical tests are included and the results demonstrate the good performance of the proposed method both for membrane and bending dominated problems.

Keywords: Shells; Triangular MITC elements; Spurious modes; Stabilization; Mixed formulations; Locking

1. Introduction

In engineering analyses shell structures are frequently encountered, hence shell finite elements are of utmost interest. As is now well established [1], one of the great challenges in the design of shell elements is to ensure reliability of the numerical procedures for all types of asymptotic behaviours. The main difficulty in this respect – which can be summarized as the ‘‘asymptotic dilemma’’ – consists in circumventing the locking phenomena arising in bending-dominated asymptotic behaviours without destroying the ability of the procedure to accurately represent membrane-dominated and mixed behaviours [2].

Some shell finite element procedures have been shown to be quite reliable for general asymptotic behaviours, and in particular the quadrilateral elements of the ‘‘MITC’’ (mixed interpolation of tensorial components) family, namely the MITC4, MITC9 and MITC16 shell elements, see [3,4]. However, in practice complex geometries must be handled, and *triangular* elements are often more adequate – or even necessary – in this respect. Some triangular MITC elements have also been proposed in [5], with detailed assessments leading to the conclusion that the 6-node MITC6a element – in particular – performs well in the various test problems considered, although some attenuated locking was observed in the (bending-dominated) axisymmetric hyperboloid test problem – especially for very small values of the thickness. Furthermore, in [6] the MITC6a element was identified as the best candidate – among a large family of possible MITC procedures – for the treatment of membrane locking, based on a simple ‘‘membrane test problem’’. However, as shown in [6], the same test problem revealed that the MITC6a element may feature some non-physical displacement modes with zero membrane energy for some combinations of midsurface geometry and boundary conditions, and additional assessment results have shown that rotation convergence is very poor for this element in membrane-dominated situations.

Nevertheless, since the MITC6a element is a most promising candidate for analyzing shell structures with triangular elements, the present paper focuses on this particular element. For the detailed formulation of the MITC6a

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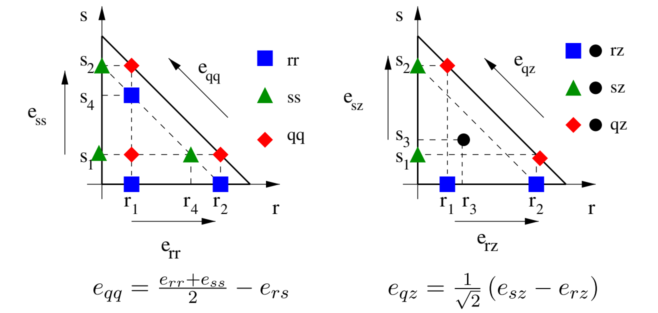

Fig. 1. MITC6a tying point locations for each covariant strain component ($r_1 = s_1 = \frac{1}{2} - \frac{1}{2\sqrt{3}}, r_2 = s_2 = \frac{1}{2} + \frac{1}{2\sqrt{3}}, r_3 = s_3 = \frac{1}{3}, r_4 = s_4 = \frac{1}{\sqrt{3}}$).

element we refer to [5], but for completeness we show in Fig. 1 the locations of the tying points used in the mixed interpolation procedure for the various strain components, see also [3,1] for general discussions and analyses pertaining to MITC procedures. The purpose of this paper is thus twofold: first to illustrate and summarize the limitations of the MITC6a element, and secondly to propose some remedies carefully designed and assessed in order to improve the membrane-dominated behaviour without deteriorating the reliability in bending-dominated and mixed situations.

The outline of the paper is as follows. In Section 2, we introduce a test problem for which the MITC6a exhibits spurious membrane energy modes and we provide detailed convergence results which show the limitations of the element. In Section 3, based on these numerical results we propose several stabilization strategies designed to improve the convergence. Then, in Section 4 we numerically assess the stabilized elements using several test problems, including that introduced in Section 2. Finally we give some conclusions.

2. The hyperbolic triangle test problem

As demonstrated in [6], for certain geometries there exist some discrete displacement fields for which the MITC6 membrane energy (shear energy not included) vanishes, although pure bending is inhibited in theory. We call these particular displacement fields "spurious membrane modes", and the aim of this section is to analyse the impact of such modes in actual solutions. Noting that spurious membranes modes *per se* are not present in the clamped hyperboloid test problem used in [5,6], we now introduce a new test problem specifically designed to investigate this issue.

We consider a shell of uniform thickness t given by the midsurface defined as the image of the triangular 2D domain of vertices $(-1,0)$, $(1,0)$ and $(0,1/2)$ by the following mapping

$$\vec{\phi}(\xi^1,\xi^2) = \begin{pmatrix} \xi^1 \\ \xi^2 \\ \dfrac{(\xi^1)^2 - (\xi^2)^2}{2} \end{pmatrix}. \tag{1}$$

The structure is clamped along the boundary corresponding to $\xi^2 = 0$, hence pure bending is inhibited (see [6]). The loading applied is given by

$$F(\vec{v}) = \int_S \mathrm{tr}\left(\underline{\underline{\gamma}}(\vec{v})\right)\mathrm{d}S, \tag{2}$$

where $\underline{\underline{\gamma}}(\vec{v})$ is the membrane strain tensor associated with displacement field $\vec{v}$ and the symbol tr denotes tensor trace. This choice of loading clearly ensures the admissibility condition for membrane dominated structures – see [1] – which is crucial to obtain a well-posed membrane-dominated problem. The additional motivation of this test problem is that the loading considered is distributed over the whole domain, hence it does not induce internal boundary layers that would entail meshing difficulties. Note that this loading corresponds to a constant isotropic membrane prestress field applied in the whole shell structure.

Throughout this paper, isotropic linearized elasticity is considered with Poisson ration $\nu = 1/3$. The actual value of Young's modulus used in the computations is of no consequence because we are only interested in *comparisons* of solutions in the assessments, not in absolute values.

Fig. 2 shows the geometry of the structure and the deformed configuration computed with displacement-based P_2 elements. Note that we resort to mesh refinement to adequately capture the boundary layer corresponding to the clamped boundary conditions, see [6]. Based on preliminary computations, the width of the refined band is taken as $3\sqrt{\varepsilon}L$, where L denotes a characteristic dimension of the structure – here, $L = 1$ – and $\varepsilon = t/L$.

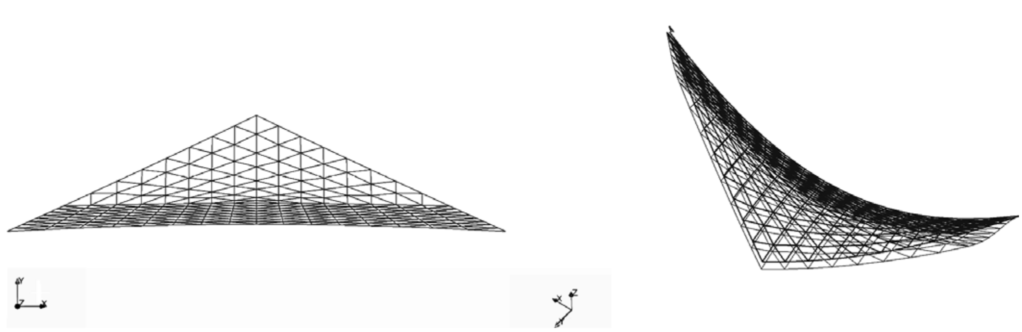

Fig. 2. Hyperbolic triangle test problem – Left: undeformed midsurface mesh with boundary refinement ($\varepsilon = 10^{-3}$); Right: deformed and undeformed meshes.