Physica A 468 (2017) 532–539

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Anomalous transports in a time-delayed system subjected to anomalous diffusion

Ru-Yin Chen, Lu-Mei Tong, Lin-Ru Nie *, Chaojie Wang, Wanli Pan

Faculty of Science, Kunming University of Science and Technology, Kunming 650500, China

HIGHLIGHTS

- Abnormal transports in an abnormal diffusion are considered here.
- Multiple current reversals by time delay are found.
- Absolute negative mobility by abnormal diffusion is reported.
- As delay time is increased, a state transition from abnormal → normal → abnormal → normal → abnormal → normal transport in the case of superdiffusion is presented.

ARTICLE INFO

Article history:
Received 29 March 2016
Received in revised form 14 September 2016
Available online 1 November 2016

Keywords:
Anomalous transport
Time delay
Anomalous diffusion
Lévy noise

ABSTRACT

We investigate anomalous transports of an inertial Brownian particle in a time-delayed periodic potential subjected to an external time-periodic force, a constant bias force, and the Lévy noise. By means of numerical calculations, effect of the time delay and the Lévy noise on its mean velocity are discussed. The results indicate that: (i) The time delay can induce both multiple current reversals (CRs) and absolute negative mobility (ANM) phenomena in the system; (ii) The CRs and ANM phenomena only take place in the region of superdiffusion, while disappear in the regions of normal diffusion; (iii) The time delay can cause state transition of the system from anomalous → normal → anomalous → normal → anomalous → normal transport in the case of superdiffusion.

1. Introduction

Anomalous transport phenomena have been explored recently, due to their wide applications to bioanalytical purposes, such as separation and fractionation of colloids, biological molecules and cells [1–3]. Essential mechanisms and conditions that the anomalous transports occur have stimulated many scientists to investigate the Brownian particles in periodic potentials. It is well known that anomalous transports consist of two categories: one is current reversal (CR), the other is absolute negative mobility (ANM). The CR, i.e., the current changes its direction in certain parameter regions of model, has been studied intensively due to its applications to modeling many different situations such as particle separation, intracellular transport and Josephson junction transport. It is known that *the CRs in ratchet systems* can be induced by noise [4–6]. Of course, CR can also occur in deterministic cases [7–10]. Apart from noise-induced CR, shape of ratchet potential also can invert direction of current [11,12]. If a potential is symmetric, temporal breaking also can make direction of current exhibit multiple reversals. For instance, two external sinusoidal forces are able to cause multiple CRs by increasing

* Corresponding author.
E-mail address: linrunie@126.com (L.-R. Nie).

http://dx.doi.org/10.1016/j.physa.2016.10.071
0378-4371/

amplitudes of the harmonic forces [13]. In addition, CR can also take place in time-delayed systems. In Ref. [14], time delay causes CR once. But multiple CRs by time delay have not been reported.

ANM, as another anomalous transport phenomenon, is gaining more and more attention, in which response of system is opposite to a small external bias force. At present, one has focused on the ANMs in normal diffusion. The authors of Ref. [15] found that a sinusoidal external driving with appropriate amplitude and frequency may lead to spontaneous symmetry breaking, and in the action of static bias force a dimer exhibits a motion opposite to that force. A Brownian particle subjected to an external time-oscillatory drive in thermal inertial ratchets moves oppositely to a constant bias force [16]. Sometimes, occurrence of ANM relies on a subtle interplay between the stability of coexisting attractors, noise-induced metastability and transient chaos [17]. With the help of time-periodic driving or space dependent damping, ANM phenomenon will be induced in spatially periodic symmetric system [18,19]. For some complex systems, diffusion processes usually no longer follow Gaussian statistics, and the Fick's second law fails to describe their transport behaviors. In anomalous diffusions, they exhibit interesting dynamical properties [20,21]. But less effort has been devoted to the ANMs in anomalous diffusions. And stability index of the Lévy noise plays a crucial role in inducing ANM phenomenon in the system considered here.

In fact, time delay cannot be neglected, and its effects are ubiquitous in nature [22–26]. In some physical systems, the time delay mainly comes from a finite transmission speed of matter and energy [27–30]. For biological systems, the time delay is attributed to information processing [31,32]. The time delay makes usually systems exhibit many interesting statistical properties [33]. Besides, in bistable systems, time delay can induce stochastic resonance [34,35] or multiresonances [36,37] in a bistable system. A time-delayed feedback can pronouncedly enhance directed transport in periodic systems [38,39]. In addition, energy conversion efficiency of Brownian motor can be improved greatly by time delay [40–42].

Indeed, anomalous diffusion is ubiquitous like time delay in the nature. In the case that both time delay and anomalous diffusion are considered simultaneously, a system becomes more realistic. In this paper, we mainly investigate anomalous transports of a Brownian particle in a ratchet periodic potential with time delay and the Lévy noise, and study effect of the time delay and the Lévy noise on CRs and ANM. The paper is constructed as follows: In Section 2, model is provided. In Section 3, results and discussions are presented. In Section 4, conclusions are made.

2. Model and theory

Here we consider motion of an inertial particle in a time-delayed periodic potential driven by a time-dependent periodic force $A\cos(\omega t)$ and a constant external driving force F. Its dynamical behavior is governed by the following equation in the dimensionless form:

$$\ddot{x}(t) + \gamma\dot{x}(t) = f(x, x_\tau) + A\cos(\omega t) + F, \tag{1}$$

where $x(t)$ is the state variable, $\dot{x}(t)$ and $\ddot{x}(t)$ are the first and second derivatives of x with respect to time t, respectively. γ is the friction coefficient, $f(x, x_\tau)$ is the spatially periodic force, with a being a constant, and $f(x, x_\tau)$ is assumed to be taken the form $f(x, x_\tau) = 2\pi\cos(2\pi x(t-\tau)) + b\pi\cos(4\pi x(t))$, where $x(t-\tau)$ represents the state variable at time $t-\tau$, with τ being the delay time. And b is the symmetry parameter of the potential, set exemplarily to $b = 0.2$, i.e., a ratchet potential for $\tau = 0$. If there exist environmental perturbations, they will give rise to a fluctuation of the spatially periodic force. Fluctuations of external parameters are expressed by a multiplicative noise [43,44]. Thus Eq. (1) can be rewritten into

$$\ddot{x}(t) + \gamma\dot{x}(t) = f(x, x_\tau) + A\cos(\omega t) + F + f(x, x_\tau)\zeta(t), \tag{2}$$

where $\zeta(t)$ is the stochastic force. Because anomalous diffusion is ubiquitous in realistic systems. we let $\zeta(t)$ be the Lévy noise to reflect the fluctuation in the system. The Lévy noise possesses three characteristic parameters of stability index α, symmetry parameter β and intensity σ. Its probability density function $L_{\alpha,\beta}(\zeta;\sigma,\mu)$ and characteristic function $\phi(\theta)$ can be given as

$$\begin{aligned}\phi(\theta) &= \int_{-\infty}^{+\infty} \mathrm{d}\zeta\, e^{i\theta\zeta} L_{\alpha,\beta}(\zeta;\sigma,\mu)\\ &= \begin{cases} \exp\left[-\sigma^\alpha|\theta|^\alpha\left(1 - i\beta\,\mathrm{sign}(\theta)\tan\dfrac{\pi\alpha}{2}\right) + i\mu\theta\right], & \alpha \neq 1 \\ \exp\left[-\sigma|\theta|\left(1 + i\beta\,\mathrm{sign}(\theta)\dfrac{2}{\pi}\ln|\theta|\right) + i\mu\theta\right], & \alpha = 1 \end{cases}\end{aligned} \tag{3}$$

where μ is the location parameter, whose value is zero when the distributions are strictly stable. Mean square displacement of $\zeta(t)$ grows like $\langle[x(t) - \langle x(t)\rangle]^2\rangle \propto t^{2/\alpha}$. When $\alpha = 2$, $\zeta(t)$ becomes normal diffusion, in contrast to superdiffusion ($\alpha < 2$) situations (i.e., Lévy flights). It will be found that the anomalous transports only appear in the region of superdiffusion, while disappear for the normal diffusion $\alpha = 2$.

Eq. (2) models the underdamped system with time delay and anomalous diffusion. Its statistical properties are incarnated by mean velocity of the particle $\langle v\rangle$. But it is very difficult to find out analytical expression of $\langle v\rangle$ in the underdamped case.