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# A PROPOSAL FOR THE THERMODYNAMICS OF CERTAIN OPEN SYSTEMS

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ABSTRACT. Motivated by the fact that the (inverse) temperature might be a function of the energy levels in the Planck distribution $n_\varepsilon = \frac{1}{\zeta^{-1}e^{\beta(\varepsilon)\varepsilon}-1}$ for the occupation number $n_\varepsilon$ of the level $\varepsilon$, we show that it can be naturally achieved by imposing the constraint concerning the conservation of a weighted sum $\sum_\varepsilon f(\varepsilon)\varepsilon n_\varepsilon$, with a fixed positive weight function $f$, of the contributions of the single energy levels occupation in the Microcanonical Ensemble scheme, obtaining $\beta(\varepsilon) \propto f(\varepsilon)$. This immediately addresses the possibility that also a weighted sum $\sum_\varepsilon g(\varepsilon)n_\varepsilon$ of the particles occupation number is conserved, having as a consequence that the chemical potential might be a function of the energy levels of the system as well. This scheme leads to a thermodynamics of open systems in the following way:

*the equilibrium is reached when the entropy function is maximised under the constraints that some weighed sums of occupation of the energy levels and the occupation numbers are conserved.*

The standard case of isolated systems corresponds to the weight functions being trivial (i.e. $f, g$ are identically 1). For such open systems, new and unexpected phenomena which might happen in nature can appear, like the Bose Einstein Condensation in excited levels. The ideas outlined in the present paper may provide a new approach for the treatment of the irreversible thermodynamics.

## 1. INTRODUCTION

The possibility that the (inverse) temperature can be a function of the energy levels of the system appeared in [1] as *Local Equilibrium* even if, perhaps, it was considered in previous studies. Recently, in [2] it has been investigated the connection of the Local Equilibrium with the principle of detailed balance for "small" open systems interacting with a "huge" reservoir.

The Local Equilibrium simply means that, in the celebrated Planck formula for the occupation numbers of Bose particles

$$n_\varepsilon = \frac{1}{\zeta^{-1}e^{\beta\varepsilon} - 1}\,, \quad \varepsilon \in \text{the set of energy levels of the system}\,, \tag{1.1}$$

the inverse temperature is supposed to be a function of $\varepsilon$: $\beta = \beta(\varepsilon)$. Here, $\zeta$ is the fugacity, and $q = 0, \pm 1$ correspond to the Boltzmann and Bose/Fermi cases.

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