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Relaxation and diffusion models with non-singular kernels

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Abstract

Anomalous relaxation and diffusion processes have been widely quantified by fractional derivative models, where the definition of the fractional-order derivative remains a historical debate due to its limitation in describing different kinds of non-exponential decays (e.g. stretched exponential decay). Meanwhile, many efforts by mathematicians and engineers have been made to overcome the singularity of power function kernel in its definition. This study first explores physical properties of relaxation and diffusion models where the temporal derivative was defined recently using an exponential kernel. Analytical analysis shows that the Caputo type derivative model with an exponential kernel cannot characterize non-exponential dynamics well-documented in anomalous relaxation and diffusion. A legitimate extension of the previous derivative is then proposed by replacing the exponential kernel with a stretched exponential kernel. Numerical tests show that the Caputo type derivative model with the stretched exponential kernel can describe a much wider range of anomalous diffusion than the exponential kernel, implying the potential applicability of the new derivative in quantifying real-world, anomalous relaxation and diffusion processes.

Key words: Anomalous relaxation and diffusion, Non-singular kernel, Stretched exponential function kernel, Memory characterization, Mean squared displacement