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A quantum anharmonic oscillator model for the stock market

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HIGHLIGHTS

- A quantum anharmonic oscillator model is proposed to describe the stock price return.
- Excited states are used to explain the leptokurtic distributions of stock indices.
- The model is applicable to both liquid and illiquid markets.
- Data filtering is applied to extract price data for the trend following behavior.

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ABSTRACT

A financially interpretable quantum model is proposed to study the probability distributions of the stock price return. The dynamics of a quantum particle is considered an analog of the motion of stock price. Then the probability distributions of price return can be computed from the wave functions that evolve according to Schrodinger equation. Instead of a harmonic oscillator in previous studies, a quantum anharmonic oscillator is applied to the stock in liquid market. The leptokurtic distributions of price return can be reproduced by our quantum model with the introduction of mixed-state and multi-potential. The trend following dominant market, in which the price return follows a bimodal distribution, is discussed as a specific case of the illiquid market.

1. Introduction

As a newly developed interdisciplinary subject applying physics theories and methods into economic studies, econophysics was first introduced in the mid-1990s, followed by a series of relevant works [1–4]. However, econophysics-like study of finance had been started much earlier. In 1900, L. Bachelier in his doctoral thesis described a dynamic model of option price using stochastic process [5], which was honored as the first piece of work in mathematical finance. Modern econophysicists view this work as an application of Brownian motion, the fundamental phenomenon of statistical physics, to the modeling of financial market. During the last two decades, different kinds of theories and methods originating from physics have been used for stock price analysis [6,7], option pricing [8,9], portfolio management [10,11], etc. Although statistical physics remains the major resource for methodologies, tools from other branches such as quantum mechanics also played an important role in the field of econophysics [8,9]. For example, B. Baaquie systematically explained the path integral method for option pricing in his book [12], while M. Schaden tried to use wave functions of cash and security to describe the state of a financial market [13].

Concurrently, another body of econophysicists concentrated on the dynamics of stock price with the help of quantum wave packets [14–17]. Ye et al. proposed a quantum harmonic oscillator model in 2008 to explain the persistent fluctuations

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in stock markets. The squared modulus of the ground state wave function was used as a probability measure of the stock price in stationary market [14]. As the study focused on the dynamics of the stock price under sudden information, no convincible financial interpretation of its Hamiltonian was addressed and one could deplore a lack of comparison between the derived probability distributions and that of the classical models. Zhang et al. explained the financial meaning of the wave function and gave a financial form of the uncertainty principle in 2010 [15]. Although they demonstrated the feasibility of quantum model by proposing a cosine formed potential, the model did not show any advantage over the quantum harmonic oscillator or the classical random walk. In this study, we propose a financially interpretable quantum Hamiltonian for stationary market and compute the probability density functions of price return from the Schrodinger equation. The potential consisting of a quadratic and a quartic term, which can be widely applied to different market environments, is derived from the dynamics of order excess in the stock market.

In the following parts of this paper, basic quantum variables and the corresponding dynamic equations (Schrodinger equation) are specified in Section 2. In Section 3, the potential operator with detailed financial interpretation for a stationary market is proposed, followed by the adjustment of parameters for liquid market. The trend following dominant market is discussed as a special case of illiquid market by both analyzing the real data and adjusting the parameters of the quantum model in Section 4. Discussion and conclusion are addressed in Section 5.

2. A quantum description of the stock market

In order to study the stylized facts of the stock price, we refer to the price return as the fractional price change

$$r(t, \Delta t) = \frac{p(t) - p(t - \Delta t)}{p(t - \Delta t)}, \tag{1}$$

with $p(t)$ the stock price at time t and Δt the time interval [18]. Assuming that the motion of a stock price is similar to that of a quantum particle, the price return can then be regarded as an analog to the position of the quantum particle. The position of a quantum particle is indeterminate and follows a probability distribution, according to Copenhagen interpretation, which is equal to the squared modulus of a wave function $\psi(r, t)$. Thus the stock price can analogously be described by a wave function of price return whose dynamics is described by Schrodinger equation as

$$\mathrm{i}\hbar \frac{\partial}{\partial t}\psi(r, t) = \boldsymbol{H}\psi(r, t), \tag{2}$$

with Hamiltonian operator

$$\boldsymbol{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial r^2} + \boldsymbol{V}(r, t), \tag{3}$$

where i is the imaginary unit, $\hbar$ is the reduced Planck constant.

The Hamiltonian operator consists of a kinetic and a potential energy part, where $\hbar$ can be regarded as the uncertainty of irrational transaction and m represents the intrinsic properties of the stock such as capital [15,17]. It shows that the state of a stock particle is influenced by both its internal properties and the external potential field. Assuming that both $\hbar$ and m are of unit value, the potential operator remains as the variable part of the model. Focusing on the stationary market environment in this study, we use a time independent potential to reduce Eq. (2) into

$$\boldsymbol{H}(r)\varphi(r) = E\varphi(r), \tag{4}$$

with $\psi(r, t) = \varphi(r)T(t)$ and $T(t) = \exp(-\mathrm{i}Et/\hbar)$, where E is a real number interpreted as the eigenenergy of the stock price and $\varphi(r)$ is the corresponding eigenfunction.

Instead of the classical consideration that the price return is of a fixed value at a definite time, in the quantum description, the price return is indeterminate before "measurement" (the finish of a deal) and follows a probability distribution according to different possible values. The probability density of price return in a stationary market is obtained from

$$\rho(r, t) = |\psi(r, t)|^2 = |\varphi(r)|^2. \tag{5}$$

3. A quantum anharmonic oscillator model

We firstly induce an equation about price return from the dynamics of order excess. The relation equation of the instantaneous price return $r(t)$ and instantaneous order excess $\Delta\phi$ is

$$r(t) = \Delta\phi/\lambda, \tag{6}$$

where $r(t) = \frac{1}{p(t)}\frac{\mathrm{d}p(t)}{\mathrm{d}t}$, $\Delta\phi = \phi_+ - \phi_-$ (ϕ_+: instantaneous demand; ϕ_-: instantaneous supply), and λ is the measurement of market depth [19].

We assume that the market consists of three types of participants: market makers, contrarians and trend followers. Market makers, working as intermediaries, absorb the existing orders by providing buy and sell quotations of the stocks

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