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Paradox of integration—A computational model

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HIGHLIGHTS

- The integration paradox by P. Blau is described mathematically for the first time.
- A sharp phase transition is identified in the space of parameters.
- The model is supplemented with a self-deprecating strategy as in the Blau theory.
- This strategy is shown to smooth the transition and improve interpersonal relations.

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ABSTRACT

The paradoxical aspect of integration of a social group has been highlighted by Blau (1964). During the integration process, the group members simultaneously compete for social status and play the role of the audience. Here we show that when the competition prevails over the desire of approval, a sharp transition breaks all friendly relations. However, as was described by Blau, people with high status are inclined to bother more with acceptance of others; this is achieved by praising others and revealing her/his own weak points. In our model, this action smooths the transition and improves interpersonal relations.

1. Introduction

More than a half of century ago, Blau described a social phenomenon which he called "paradox of integration" [1]. According to Blau, 'social integration is a dense network of friendships' [2]. Further, an integration process of a social group includes two competing processes: attempts to appear attractive raise both attraction and repulsion [1]. While the former reaction is natural, the latter comes from the fear of being dominated. As a paradoxical consequence, most attractive persons can be rejected by the group. Having this in mind, persons both attractive and smart maintain their popularity by self-mockery and praising others. Up to our knowledge the effect remains unnoticed by social modellers, despite its importance as of a collective social phenomenon.

Taking the Blau description as granted, we intend to sharpen the picture of the paradox by developing its quantitative aspects. There is vast literature about dangers of quantitative social modelling, provided by both sociologists and modellers themselves [3–5]. Taking this into account, we are more attached to the internal logic of the social phenomenon, provided by the model, than to the calculated values of the model variables. We believe that the quantitative research should provide scenarios based on hypothetical "what if" assumptions. Below, attempts to attain higher status at expense of somebody else will be encoded symbolically as 'critique', and attempts to reach sympathy—as 'praising'. We are going to consider two versions of the model, without and with the self-deprecating strategy. Although we do not expect this strategy to be absent in real societies, we hope that the counterfactual approach allows to identify its consequences.

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According to both scenarios considered here, an agent i praises or critiques another agent j, losing or gaining her/his own status, respectively (for simplicity we write on males from now on). Simultaneously, the status of a praised agent j increases, and the status of a criticized agent j is reduced. These variations are balanced, however, by the acceptance of i by the praised agent j, altogether with the acceptance of i by those with the same status as j. On the contrary, a critique raises hostility towards i of the criticized agent j as well as of those with the same status as j. In the second scenario, agents adopt also the self-deprecating strategy. Then, their utility functions depend additionally on their actual status: if one's need of high status is already fulfilled, an agent is more prone to praise others [1]. A reverse of this strategy is the shame-rage spiral [6]; own status perceived as low is known to trigger aggression.

The goal of this work is to capture the collective character of the phenomenon. Coupling between agents is due to the variations of interpersonal relations, which involve all agents of the same status as the one who is praised or criticized. (A similar solidarity has been suggested by assuming that agents of the same size are prone to cooperate [7].) This group reaction weakens, however, due to a creeping polarization with respect to status. Social polarization known to be ubiquitous [8,9]; its relation to our work is limited to the process of status formation in groups. Yet, even this limited aspect of the polarization is essential for the formation of group structure, group perception and identity formation [10,11]. Although the polarization is not our founding assumption, it appears as a natural consequence of the modelled dynamics.

In the next section, the algorithm is described in details. Sections 3 and 4 are devoted to our numerical results and their discussion, respectively. In the last section we note that the term "paradox of integration" has been used recently in a different meaning, and we discuss the mutual relation of these two phenomena.

2. Algorithm

In a fully connected network of N nodes, an agent is represented by a node. A social status A_i is assigned to each agent i; initial values of those variables are small integers, selected randomly to be zero or ± 1. Here, integer representation is chosen for its simplicity. The relations between agents are encoded in the form of an asymmetric matrix $x(i,j) = \pm 1$, with elements $+1$ (friendly) or -1 (hostile). The matrix element $x(i,j)$ specifies the relation of i towards j.

In the first scenario, the simpler one out of the two considered here, the evolution proceeds as follows. At each time step, an ordered pair (i,j) of different agents is selected randomly. (This means, that we select an agent i and next we select an agent j, with the probabilities $1/N$ and $1/(N-1)$ respectively, the same for all agents.) The ith agent evaluates his utility function $f(i,j)$ if he praises or critiques the jth agent. To do this, he needs to know the number $\upsilon(A_j)$ of agents with the same status as the jth agent, including j himself. The decision – to praise or to critique – is taken by checking the sign of $f(i,j)$ given by

$$f(i,j) = -p + \frac{1-p}{N-1}\upsilon(A_j). \qquad (1)$$

Here, p is the weight of the preference of status, and $1-p$ is the weight of the preference of acceptance. Once the former prevails, i.e. $f(i,j) < 0$, the decision is to critique. Then, $x(k,i)$ is set to be -1 for all agents k such that $A_k = A_j$. Next, the status A_i is increased by one and the status A_j is reduced by one; note that the change of status concerns only two agents. In the opposite case, when $f(i,j) > 0$, the decision is to praise. Then, all changes go quite the opposite: $x(k,i)$ is set to be $+1$ for all agents k such that $A_k = A_j$, next $A_i \to A_i - 1$ and $A_j \to A_j + 1$. In this way, the mean value of the status is kept constant within the model; each increase of A_j is accompanied by a decrease of A_i, and the opposite. This makes the status similar to IQ, with its mean value assumed to be 100 for each society. Actually, only relative variations of status are relevant.

The second scenario is all the same; the only modification is that the agents' decisions include the dependence of the weight p on the actual status of the decision-maker. As explained above, the mechanism is that p decreases with A_i. Here we redefine the parameter p as

$$p'_i = \frac{2p}{1+2^{A_i}} \qquad (2)$$

what keeps $p'_i < 1$ as long as $p < 0.5$. This function captures both effects: the self-deprecating strategy and the shame-rage spiral.

3. Results

For both scenarios, we trace the mean value $\langle x \rangle$ of all relations x_{ij} against time t, averaged also over 100 realizations. This is an indicator of the kind of relations: friendly (positive) or hostile (negative). Also, keeping in mind that the mean value of the status remains constant, we are interested in its standard deviation. Initial values of the relations $x(i,j)$ do not influence the results, as they are forgotten in a few time steps. The model dynamics describes the process of differentiation of status; then it makes sense to assume a narrow distribution of the initial values of A_i, as we do.

The results on $\langle x \rangle$ obtained within the first scenario, where the weight p is constant, are shown in Fig. 1. It appears that the behaviour of $\langle x \rangle$ sharply depends on the parameter p, and this dependence manifests only after some transient time. There is a critical value of p, denoted as p_c from now on; for $p < p_c$, the mean relation tends to $+1$, while for $p > p_c$ it goes