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Pricing credit default swaps under a multi-scale stochastic volatility model

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ABSTRACT

In this paper, we consider the pricing of credit default swaps (CDSs) with the reference asset driven by a geometric Brownian motion with a multi-scale stochastic volatility (SV), which is a two-factor volatility process with one factor controlling the fast time scale and the other representing the slow time scale. A key feature of the current methodology is to establish an equivalence relationship between the CDS and the down-and-out binary option through the discussion of "no default" probability, while balancing the two SV processes with the perturbation method. An approximate but closed-form pricing formula for the CDS contract is finally obtained, whose accuracy is in the order of $\mathcal{O}(\epsilon + \delta + \sqrt{\epsilon\delta})$.

1. Introduction

How to effectively manage and control credit risks is a hot topic in today's financial engineering area, because this kind of risks is one of the most adverse factors for the development of financial markets and is also the primary cause of financial crises. As a new kind of financial instruments, credit derivatives are nowadays playing a significant role in dealing with problems caused by credit risks. Among them, the most basic and successful one is the so-called credit default swap (CDS).

A CDS is a contract that allows credit risks to be traded. In specific, the buyer of the CDS pays a regular fee to the seller until the end of the contract or until a credit event occurs, whereas the seller of this contract undertakes the responsibility of compensating the buyer in case of default. Through this kind of trading mechanics, it is clear that the CDS is able to transfer credit risks from its buyer to the seller.

The accurate determination of the CDS price is fundamental for financial institutions, because it cannot only help those institutions to determine capital reserves to set aside to cover for risk connected to financing and investment activities, but also mitigate credit exposures through hedging them with credit derivatives, such as the CDS. Choosing suitable models for credit risks and the reference asset is crucial in the accurate pricing of CDS. In the literature, the credit risks are usually modelled by two kinds of models, i.e., the reduced-form models and the structural models. The formers are adopted by a number of researchers including Duffie & Singleton [1], Jarrow et al. [2] and Hull & White [3]. With the flexibility in the functional form, these models are able to provide strong in-sample fitting properties. However, they fail to capture the wide range of default correlations and may result in poor out-of-sample behaviours, as suggested by several empirical studies [4,5]. The structural models, as another alternative, use the evolution of the reference asset and the value of the debt to determine the probability of default. Typical models in this category include the Merton model [6], which characterizes the breach of default by assuming that the default would occur if the company is insolvent. Although elegant, some of the

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assumptions under this model are unrealistic. For example, under this model, it is assumed that the target company would only default at the expiry date and the value of the company can drop to almost zero without default. However, nowadays, the default of a company can be triggered when its value is below a certain level away from zero at any time before or at the maturity of the bond.

As far as the modelling of the evolution of the reference asset is concerned, Merton [6] assumes that it follows a geometric Brownian motion. This assumption is also adopted by the classical Black–Scholes (B–S) model [7] for the underlying asset. Empirical studies have, however, suggested that this assumption is at odds with some of the real market conditions [8,9], and usually leads to the mispricing of financial derivatives. There are a number of modifications to such an assumption and some of them have already been used in the CDS pricing field. For example, de Malherbe [10] replaced the geometric Brownian motion by a Poisson process, and determined the corresponding CDS price by a probabilistic approach. With stochastic intensity models adopted for the default events, Brigo and Chourdakis [11] considered the pricing of CDS when the counterparty risk is also taken into consideration. Recently, He & Chen [12] adopted the generalized mixed fractional Brownian motion for the reference asset and derived a closed-form formula for the price of the CDS. However, in most of the work mentioned above, the default of the company is assumed to be triggered only at the maturity date.

In this paper, we shall replace the constant volatility appearing in the Merton model by a stochastic volatility driven by two time scales. This so-called multi-scale stochastic volatility (SV) model has many advantages over a single time scale and is much closer to the real financial market conditions because it cannot only capture the long range memory characteristic of the volatility correlations but also ensures the leverage effect to decay much faster than the volatility correlation [13,14]. Our solution process begins by deriving an analytical expression for the CDS price under a general default model, in which the "no default" probability still needs to be determined. A key step of the solution process is to establish an equivalence relationship between the unknown probability and the down-and-out binary option. With the perturbation method as used in Ref. [13], a sequence of simplified systems governing the price of the down-and-out binary option are obtained and solved. The price of the CDS is then obtained.

The rest of the paper is organized as follows. In Section 2, the multi-scale SV models are reviewed. In Section 3, the CDS contract considered in this paper is specified and the general expression for the fair price of the CDS is derived. After that, the partial differential equation (PDE) system governing the key part of the CDS price is established, based on which the approximation solution is derived by the perturbation methods. Concluding remarks are given in the last section.

2. Multi-scale volatility models

In this section, the multi-scale SV models are briefly revisited for the sake of completeness of the paper. This kind of models are introduced by Fouque et al. [13,15] based upon various empirical studies. Under these models, the underlying S_t is assumed to follow a geometric Brownian motion with SV controlled by a fast and a slow time scale. In specific, S_t satisfies

$$\frac{\mathrm{d}S_t}{S_t} = \mu \mathrm{d}t + f(Y_t, Z_t)\mathrm{d}B_{1,t},$$

where μ is the drift rate, $B_{1,t}$ is a standard Brownian motion, and f is a bounded positive function representing the SV. Moreover, f is driven by two other factors, Y_t and Z_t, which are governed by the following two processes as

$$\mathrm{d}Y_t = \left[\frac{1}{\epsilon}(m - Y_t)\right]\mathrm{d}t + \frac{\sqrt{2}v}{\sqrt{\epsilon}}\mathrm{d}B_{2,t},$$
$$\mathrm{d}Z_t = \delta c(Z_t) + \sqrt{\delta} g(Z_t)\mathrm{d}B_{3,t},$$

where the functions $c(z)$ and $g(z)$ are smooth and grow at most linearly as $z \to \infty$. v^2 represents the variance of the invariant distribution of Y_t. It determines the long-run level of the volatility fluctuations. Moreover, $\frac{1}{\epsilon}$ is the mean-reverting rate of Y_t controlling the reversion speed to the long-term mean m. By assuming that ϵ is a positive small parameter, Y_t is referred to as the fast volatility factor because its autocorrelation now decays exponentially on the time scale ϵ. On the other hand, for the process Z_t, it is assumed that δ is also a small positive parameter, and Z_t is referred to as the slow volatility factor. We remark that the independence of ϵ and δ is consistent with market observations [16]. Recent studies also suggest that ϵ and δ can be different by an order of magnitude (roughly $\epsilon \backsim \mathcal{O}(0.005)$ and $\delta \backsim \mathcal{O}(0.05)$). Therefore, following a major assumption made in Refs. [15,13,14], we shall focus on the case of $\epsilon \neq \delta$ in the current paper. It should also be remarked that the three Brownian motions are not necessarily independent, and the correlation among them can be expressed as

$$\mathbf{B_t} = \begin{pmatrix} 1 & 0 & 0 \\ \rho_1 & \sqrt{1-\rho_1^2} & 0 \\ \rho_2 & \rho_3 & \sqrt{1-\rho_2^2-\rho_3^2} \end{pmatrix} \mathbf{W_t},$$

where $\mathbf{B_t} = \begin{pmatrix} B_{1,t} \\ B_{2,t} \\ B_{3,t} \end{pmatrix}$ and $\mathbf{W_t}$ is a standard three-dimensional Brownian motion.