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Masonry and monolithic circular arches strengthened with composite materials – A finite element model

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ABSTRACT

A finite element model for the stress analysis of circular arches strengthened with composite materials is developed. The formulation uses the principle of virtual work, the Bernoulli–Euler curved beam theory for the arch and the composite reinforcement, and a high-order kinematic assumption that satisfies the compatibility and (with the constitutive laws) the tangential equilibrium conditions of the adhesive. The character of the masonry arch is introduced through the constitutive equations with a distinction between the masonry units and the mortar joints. Convergence and numerical studies that support using high-order shape functions and examine the capabilities of the model are presented.

1. Introduction

The use of building materials with low or zero tensile strength has defined the demand for a special structural geometry that can support the structure's own weight and resist loads through compressive stresses only. Arch structures, which strictly follow this concept, are found in almost every historic and modern built environment around the world. Due to the sensitivity of the arch to the live load to dead load ratio, the deterioration of the building materials, and the modern demands for strength and functionality, strengthening of the existing structure is often required. The use of externally bonded composite materials and mainly fiber reinforced polymer (FRP) strips for the strengthening task is advantageous in terms of superior mechanical properties, low mass, minimal dimensions, ease of installation, geometrical versatility, and improved durability. In particular, the ability to provide the arch with an adhesively bonded layer of external reinforcement that can be easily adjusted to the unique geometry of the structure is a major advantage.

The analysis of the externally strengthened arch and the variety of physical phenomena that govern its response set a notable analytical and computational challenge. Within this challenge, a distinction is made between the "stress analysis" of the masonry structure and its "limit analysis". The first challenge, which is the subject matter of this paper, aims to determine the displacements and the stresses under a given level and pattern of loads or tractions. The second challenge, which is not addressed here, aims to assess the collapse (ultimate) load of the arch. The limit analysis usually uses plastic theorems to determine an upper bound collapse load, see for example Caporale et al. [1], Crisfield and Packham [2], Heyman [3], Drosopoulos et al. [4]. As clarified by Crisfield and Packham [2], four hinges are postulated and the collapse load is computed by means of the principle of virtual work of the mechanism. By investigating all possible hinge configurations, one can obtain the lowest of the "upper bound" solutions. The resulting thrust line should not pass outside the arch, thus plasticity theory can be used to argue that a "safe" or "lower bound" load is obtained (Heyman [3], Crisfield and Packham [2], Cavicchi and Gambarotta [5], Ricamato [6]). In that sense, the limit analysis is fundamentally different from the stress analysis, it has different objectives, and it applies different analytical and theoretical concepts. It should, however, be noted that a comprehensive nonlinear stress analysis that takes the effects of cracking and accumulation of damage into account, simulates the evolution of the failure mechanism, and applies a set of failure criteria can provide insight on the ultimate limit state (collapse) behavior of the structure. The present paper aims to take a step towards the development of a FE platform for the stress analysis of the strengthened masonry arch. This step can then be further augmented to consider the deep nonlinear behavior that evolves toward the collapse load of the strengthened arch (yet, this aspect is not directly addressed here).

Analytical models for the stress analysis of the strengthened arch were presented by Valluzzi and Modena [7], Valluzzi et al. [8], and Chen [9]. In these models, a linear strain distribution

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through the entire depth of the strengthened cross section was assumed. This family of models well characterizes the global response of the arch. However, it does not take into account the direct effect of the interfacial peeling stresses due to the stress/deformations pattern in the adhesive layer and their effect on the localized response near irregular points, cracks, edges, joints, debonded regions etc. An analytical model for the stress analysis of a masonry arch comprised of three monolithic elements connected by hinges was presented by Foraboschi [10]. Also here, the effect of the high-order stress patterns in the adhesive layer on the interfacial stresses was not taken into account. A first order stress analysis model with uniform stresses through the depth of the adhesive layer in FRP plated curved members was presented by De Lorenzis et al. [11,12]. Marfia et al. [13] presented a veriational stress analysis model for the reinforced masonry arch and used the assumption of linear strain distribution through the entire strengthened cross section along with a nonlinear constitutive relation for the masonry material. The use of the assumption on the strain distribution allows for the consideration of the overall behavior but it limits the consideration of the interfacial stress concentrations near cracks, edges, joints, interfaces, etc.

An analytical model for stress analysis of FRP strengthened *monolithic* arches was presented by Elmalich and Rabinovitch [14]. This model, which was formulated as a set of differential equations in terms of displacement unknowns (strong form), addressed the challenge of describing the high-order stress field though the depth of the adhesive layer and the corresponding local behavior near the edges of the FRP strip. The direct solution of the governing equations of the model using standard boundary value problem (BVP) solvers is, however, usually limited to small-scale structures. This limitation, which mainly results from the combination of different length scales, makes the analysis of realistic arches a computational effort demanding challenge. In some cases, an analytical type of solution of the set of governing equations in [14] may also apply (see for example [15] for strengthened RC beams and [16] for curved sandwich panels) but the extension of such analytical solution to the nonlinear case is not possible. Finally, the application of the strong form approach [14] to a general structural analysis that combines various segments (strengthened on the intrados, or the extrados, unstrengthened segments) other structural components (beams, columns, etc.) or to the analysis of masonry arches is rather limited.

Finite element (FE) linear and nonlinear analysis is among the leading tools in modern structural analysis. Luciano et al. [17] used FE analyses to study a strengthened arch made of six voussoirs (masonry units). Four different constitutive models were adopted for the masonry, a linear elastic model was adopted for the FRP strip, and a no-tension model was used for the interfaces. In the plane stress 2D analysis, the voussoirs were modeled using four quadrilateral elements, the joints between two adjacent voussoirs were modeled using four-node no-tension elements, and the FRP strips were modeled using two-node truss elements. The adhesive layer and its unique stress field were not considered assuming full bond between the FRP strip and the arch. Lourenco et al. [18] used 4 eight-node elements to model the voussoirs, six-node elements to simulate the mortar and the adhesive layer, and a three-node cable element to simulate the FRP layer. Although this type of modeling can take the interfacial effects into account, the modeling ends up with a rough resolution in which the local behavior is difficult to quantify. In addition, the singularities near irregular points significantly affect the ability to predict the shear stress gradients and the radial normal stresses, which may lead to the initiation of a detachment failure of the FRP layer.

In spite of its advantages, the standard finite element modeling of the FRP strengthened arch is rather problematic. Finite element analyses of reinforced concrete beams strengthened with FRP strips (e.g. [19–21]) showed that at least 2–3 elements through the height of the adhesive layer (and in many cases even more) are required in order to consider the localized effects, the stress distribution through the adhesive layer, and the interfacial stresses. This demand ends up with an enormous size of the computational problem. For example, in standard 2D FE analysis of a 4 m span semi circular arch ($r = 2$ m) strengthened with an FRP system bonded on the intrados with a 5 mm thick adhesive layer, the number of DOF in the strengthening system itself (adhesive layer + FRP layer) is estimated about 50,000. In the entire strengthened arch, the total number of DOFs may grow to hundreds of thousands. The notable differences in the elastic properties between the components also contribute to the computational complexity. Furthermore, in some cases, the standard FE analysis does not satisfy the free edge boundary conditions, and the normal stresses at the edge of the adhesive–FRP and adhesive–arch interfaces tend to diverge. These observations imply that the stress analysis of the strengthened arch using standard FE tools is problematic and an alternative approach that uses a specific FE formulation is needed.

The objectives of this paper are to develop a FE model for the stress analysis of monolithic and masonry arches strengthened with externally bonded composite materials. The model aims to combine the advantages of the high-order modeling approach of the strengthened arch [14] with those of the finite element method and thus to support the local and overall stress analysis of full scale complicated masonry structures such as the strengthened masonry arch.

The modeling assumptions follow [14] and adopt the Bernoulli–Euler curved beam theory with small displacements for the independent consideration of the arch and the FRP strips. The adhesive layer is considered as a 2D elastic medium with resistance to shear and radial normal stresses. The tangential stiffness of the adhesive layer, which is about two orders of magnitude smaller than the tangential stiffness of the adjacent components, is neglected (also see [21]). The displacement fields and the stress fields in the adhesive layer adopt the functional form of the high-order displacements formulation of [14]. It is assumed that the adhesive–arch and the adhesive–FRP interfaces are fully bonded and that the external loads are exerted at the centroid of the arch only. It is also assumed that stresses and the displacements are uniform through the width of each component. The augmentation of the model to masonry arches assumes that cracking is limited to the mortar joints only [22] and introduces this effect through a no-tension type of constitutive model for the mortar joints. It is further assumed that once the mortar joint is cracked, the increased strains in the FRP reinforcement bridging the crack and the inability of the cracked faces to transfer shear stresses trigger the formation of a debonded region (see Hamilton and Dolan [22], Hamed and Rabinovitch [23,24]). In the debonded region, which usually extends through the joint, the adhesive looses its ability to transfer shear stresses. In many cases, and especially in the case of pre-existing cracks, the adhesive in the cracked region fails to transfer normal stresses as well. For brevity, the mathematical formulation presented next focuses on perfectly bonded segments whereas the effect of debonding is approximately introduced through the control of the properties of the adhesive material.

Falling within the category of stress analysis models, the FE analysis developed in this paper aims to describe the structural response of the strengthened arch under different levels of loads and under the assumptions mentioned above. The first step taken here towards the development of a FE framework does not directly take the collapse range, the failure criteria, or, alternatively, the application of plastic theorems to various collapse mechanisms into account. Due to the above limitations, the model does not aim to predict the collapse load of the arch but to provide a structural characterization of its response under various levels of load. The augmentation of the model to release the above limitations and