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A new statistic to capture the level dependence in stock price volatility

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ABSTRACT

In this paper, we propose a new covariance estimator based on daily opening, high, low and closing prices. We prove theoretically that the new estimator is unbiased for a pure random walk and further validate it with simulation studies. However, upon examining empirically four indices namely: NIFTY, S&P500, FTSE100 and DAX over the sample period from January 1996 to March 2015, we find that the estimator is upward biased for all the indices under study. This overreaction in stock indices can be attributed to the level dependence in stock indices, something that is not captured by the random walk model. So we explore an alternative to random walk, namely: Constant Elasticity of Variance (CEV) specification. Simulation studies provide supporting evidence that the CEV specification can capture the level dependence that makes the estimator upward biased as seen in the data. Therefore, through this specification exercise, we can see that it is possible to isolate the effect of intraday level dependence in stock prices using our estimator.

1. Introduction

Volatility estimation has always been an important aspect in pricing and hedging of assets. Starting with the assumption of constant volatility in the Black–Scholes model of option pricing, we have come a long way where we have acknowledged that volatility is not constant, but a stochastic variable. This area of research has attracted a lot of attention, giving rise to studies of alternative estimators of volatility. While the initial focus of most of these studies was on using the daily closing price alone of asset returns for volatility estimation, many studies like Alizadeh, Brandt, and Diebold (2002), Ball and Torous (1984), Brandt and Jones (2006), Garman and Klass (1980) and Parkinson (1980) have come up with efficient estimators of volatility based on high-low prices. Most of the studies assume that log-prices follow a geometric Brownian motion with or without a drift term. Such a range-based estimation for volatility using opening, high, low and closing prices of assets have also been used by many authors like Kunitomo (1992), Magdon-Ismail and Atiya (2003), Rogers and Satchell (1991) and Yang and Zhang (2000). Shu and Zhang (2006) have compared the relative performance of the four range based volatility estimators including Parkinson, Garman–Klass, Rogers–Satchell, and Yang–Zhang estimators for the S&P500 index. They found that the estimators are efficient only when the assumption of a geometric Brownian motion for the price of the underlying asset is maintained.

When it comes to estimating correlation or covariance, the most classical approach is based on estimating correlation from daily returns using standard sample correlation formula. However, Zimmerman, Zumbo, and Williams (2003) proved that this estimator is biased depending on the underlying distribution. Rogers and Shepp (2006) proposed a method to compute the range correlation from linear combination of the four intraday prices namely: open, high, low and close for two correlated Brownian motions for two stocks. Brandt and Diebold (2006) and Brunetti and Lildholdt (2002) looked at estimating the covariance of foreign exchange rates. One of the major drawbacks of all these studies were that they assumed Brownian motions as their underlying specification. This restricted the applicability of the estimators they developed. Further, the above mentioned papers provide estimators of the covariance only by making use of the assumption of no arbitrage. To put it differently, the solution they provide is too specific to the assets that are under consideration such as foreign exchange rates. That is to say, there is no statistical advance in these papers that can be *universally* applied.

Rogers and Zhou (2008) are the pioneers in developing a new method of estimating the correlation of a pair of correlated Brownian motions based on the daily opening, high, low and closing prices

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of stocks. However, their estimator suffers from the drawback that it is derived under a Brownian motion assumption. Furthermore, it really does not make sense to look for an unbiased estimator of the correlation when under the null hypothesis $\rho = +1$ because the estimate $\hat{\rho}$ can never be more than 1. Therefore, to be unbiased, the estimate $\hat{\rho}$ would always have to be 1. Similarly even with $\rho = -1$, the same thing is true because $\hat{\rho}$ cannot be less than −1.

Many studies further extended the Roger's and Zhou estimator. Sepp (2011) investigated the problem of modelling correlation of stock returns to understand the volatility of index. They chose a single factor model with mean reversion under the assumption that all correlations are equal. They found that a mean reversion term improves the predictive capacity of the model. Further Park (2014) suggested a way to improve the portfolio performance using range correlation from daily open, high, low and close data. They build portfolios based on these range correlations and investigated the performance using Sharpe Ratio. They found that if portfolio risks show lower level with no change in returns, the performance can be improved using range correlation.

Thus, from the literature we can see that there have been very few studies that aimed to find a covariance estimator (as opposed to a correlation estimator) based on intraday opening, high, low and closing prices that are unbiased. This gap motivated us to propose a new unbiased estimator of covariance. The covariance estimator based on high and low prices introduced in this paper (as explained earlier) is closely related to the work done by Rogers and Zhou (2008). Our contribution in this paper is to propose an unbiased covariance estimator based on the high-low prices, where it needs to be noted that the covariance does not have to lie in any particular range. Moreover, we make the simplest underlying assumption of a random walk thereby making the estimator very generic.

In this paper, we aim to do a specification test to understand the true stochastic process of the assets under consideration. For this, we propose a new covariance estimator *Cov Ratio* which is more general in nature thus making it universally applicable and we are first interested in proving that the proposed estimator is unbiased under the random walk assumption. We find that the estimator proposed is theoretically unbiased. This result is further strengthened with the help of simulation studies under the random walk assumption. However, when we empirically test four major stock indices: NIFTY, S&P500, FTSE100 and DAX, we find that the estimator is generally upward biased for all the indices.

This proves that the indices under consideration do not follow a random walk as evidenced by the upward bias in the estimator. We conjecture that there exists evidence of level dependence in stock price volatility that makes the estimator biased, something that is not captured by the random walk. Level dependence in volatility of stock price means that there exists a relation between level of asset price and volatility. The intraday local volatility of stock price is impacted by intraday level of stock prices. Therefore we explore an alternative to random walk under which such a bias can arise. So, we go for the next alternative, wherein we introduce the Constant Elasticity of Variance (CEV) specification to capture the level dependence. The CEV specification developed by Cox (1996) explained the inverse relation between stock price and volatility. Using a modified model of the CEV specification for the purpose of our study, we capture the upward bias seen in the data based on simulation exercise.

The rest of the paper is organized as follows. Section 2 contains the methodology in which we provide the theoretical proof of the unbiasedness of the proposed estimator under the assumption of a random walk. We also introduce the new covariance estimator *Cov Ratio*. Section 3 deals with the simulation analysis undertaken. The first part of this section deals with the simulation study for a random walk. In the second part of the section, we introduce the CEV specification. We briefly introduce the model and review the past studies related to CEV. We also carry out simulation studies under the CEV specification. Section 4 covers the empirical findings. The last section concludes the paper.

2. Methodology

In this section, we derive the mathematical proof to show that the proposed estimator is unbiased for a pure random walk under the duality concept.

Consider a pure random walk.

Assume X_i are iid following any distribution

$$S_0 = 0;$$
$$S_1 = x_1$$
$$S_2 = x_1 + x_2$$
$$S_n = x_1 + x_2 + x_3 + \ldots + x_n;$$

$$S_N = x_1 + x_2 + x_3 + \ldots + x_N; \tag{1}$$

We now define (for the ease of notation):

$x = S_N$; where S_N is the terminal value of random walk path.

$b = M_N$; where $M_N = \max\{S_n : 0 \le n \le N\}$ is the maximum of the random walk path.

$c = m_N$; where $m_N = \min\{S_n : 0 \le n \le N\}$ is the minimum of the random walk path.

By the duality of random walk which induces the time reversibility in the process; we can define:

$$S_n{}^* = S_N - S_{N-n} \quad \forall\, 0 \le n \le N \tag{2}$$

When n = 0; $S_n{}^* = S_N - S_{N-n} \rightarrow S_0{}^* = 0.$

When n = 1; $S_n{}^* = S_N - S_{N-1} \rightarrow S_1{}^* = x_N$

When n = 2; $S_n{}^* = S_N - S_{N-2} \rightarrow S_2{}^* = x_N + x_{N-1}$

When n = N; $S_n{}^* = S_N - S_{N-N} \rightarrow S_N{}^* = S_N = x_N + x_{N-1} + \ldots + x_2 + x_1$ (3)

From Eqs. (1) and (3), in terms of **joint distribution**:

$$\left\{S_n : 0 \le n \le N\right\} \text{ is distributed same as } \left\{S_n{}^* : 0 \le n \le N\right\} \tag{3.a}$$

Result 2.1: $S_N{}^* = S_N$

Let us define: $M_N{}^* = \max\{S_n{}^* : 0 \le n \le N\}$

From (2), we can rewrite:

$$M_N{}^* = \max\left\{S_N - S_{N-n} : 0 \le \text{n} \le \text{N}\right\}$$
$$M_N{}^* = S_N - \min\left\{S_{N-n} : 0 \le n \le N\right\}$$

Recall: $m_N = \min\left\{S_n : 0 \le n \le N\right\}$.

Thus by substituting, we will get:

Result 2.2: $M_N{}^* = S_N - m_N$

(3.a) implies that

$$\left\{S_N, M_N\right\} \text{ is distributed same as} \{S_N{}^*, M_N{}^*\} \tag{3.b}$$

We also know $M_N{}^*$ **is distributed same as** M_N

From Result 2.1 and 2.2, in terms of the **joint distribution**:

$$\{S_N{}^*, M_N{}^*\} = \left\{S_N, S_N - m_N\right\} \tag{4}$$

LHS product and expectation of Eq. (4) is: $\mathrm{E}[S_N{}^* M_N{}^*] = \mathrm{E}[S_N M_N]$

RHS product and expectation of Eq. (4) is: $\mathrm{E}[S_N (S_N - m_N)]$

On equating LHS and RHS, we get:

$$\mathrm{E}[S_N M_N] = \mathrm{E}[S_N (S_N - m_N)]$$

$$E[S_N M_N] = \mathrm{E}\left[S_N{}^2\right] - E[S_N m_N]$$

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