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An efficient method for evaluating the natural frequencies of structures with uncertain-but-bounded parameters

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ABSTRACT

Using interval theory and the second-order Taylor series, the eigenvalue problems of structures with multi-parameter can be transformed into those with single parameter. The epsilon-algorithm is used to accelerate the convergence of the Neumann series to obtain the bounds of eigenvalues of structures with single interval parameter, thus increasing the computing accuracy and reducing the computational effort. Finally, the effect of uncertain parameters on natural frequencies is evaluated. Two engineering examples show that the proposed method can give better results than those obtained by the first-order approximation, even if the uncertainties of parameters are fairly large.

1. Introduction

The natural frequency analysis for structures with deterministic parameters has been extensively developed. However, in engineering situations, the structural parameters are often uncertain, such as the inaccuracy of the measurement, errors in the manufacturing and assembly process, invalidity of some components and uncertainty in boundary conditions etc. The uncertainties of structural parameters may lead to large and unexpected excursion of responses that may lead to drastic reduction in accuracy and precision of the operation. Therefore, uncertainty plays an important role in the modern engineering structural analysis.

Over the past decades, a number of methods have been developed that include uncertain model properties in the finite element (FE) analysis and aim at the quantification of the uncertainty on the analysis result. The probabilistic concept is by far the most popular method for numerical uncertainty modeling. Its popularity has led to a number of probabilistic FE procedures [1–3]. However, probabilistic modeling is not the only way to describe the uncertainty, and uncertainty is not equal to randomness. Indeed, the probabilistic approaches are not able to deliver reliable results at the required precision without sufficient experimental data to validate regarding the joint probability densities of the random variables or functions involved. Therefore, one may recognize that uncertainties in parameters can be modeled on the basis of alterative, non-probabilistic conceptual frameworks. One such approach, based on a set theoretic formulation, is an unknown-but-bounded model (convex models). Such set models of uncertainty have been applied by Deif in linear programming and system theory [4]. Recently, such set models of uncertainties in parameters have drawn interest both from the system control robustness analysis field and from the structural failure measures field. For example, the convex model was introduced by Ben-Haim and Elishakoff [5], discussed later by Lindberg [6], for the study of dynamic response and failure of structures under pulsed parametric loading; the convex model has been applied in determining the upper and lower bounds of static response for structures by Liu et al. [7]. The convex model has also been applied to impulsive response, buckling analysis and optimal design of structures with uncertain parameters [8–11].

Since the mid-1960s, a new method called the interval analysis has appeared. Moore [12], Alefeld and Herzberger [13] have done the pioneering work. Mathematically, linear interval equations, nonlinear interval equation, and interval eigenvalue problems have been partially resolved. But because of the complexity of the algorithm, it is difficult to apply these results to practical engineering problems. Recently, the interval finite element (IFE) method was presented by Chen et al. [14] which makes the method easier to deal with the interval eigenvalues for closed-loop systems of structures with uncertain parameters.

It should be pointed out that the previous interval analysis methods for computing the upper and lower bounds of response of structures with interval parameters are based on the first-order perturbation or the first-order Taylor series. Although the procedure is easy to implement on the computer and incorporates the finite element code, the applications of the previous methods are limited to the case when the interval uncertainties of parameters are small. However, if the parameter uncertainties are fairly large or the combination of a large number uncertain parameters, the

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accuracy of the computational results will become unacceptable. Thus, it is highly desirable to present a more accurate method for computing the upper and lower bounds of responses of structures with fairly large uncertainties of interval parameters.

To this end, this paper presents an efficient method to estimate the natural frequencies of structures for the case with fairly large uncertainties of parameters. As we known, the eigenvalues λ is defined as ω^2 in mode analysis of FEM problems, where ω denotes the natural frequencies of structures. The idea of the proposed method is that the eigenvalues are considered as functions of the structural parameters; using interval theory and the second-order Taylor series expansion, we transform the eigenvalue problems of structures with multi-parameter into those with single parameter; The epsilon-algorithm is used to accelerate the convergence of Neumann series to compute the eigenvalues of structures with single interval parameter, thus the interval eigenvalues of the structures with multi-parameter are obtained and the effect of uncertain parameters on natural frequencies is evaluated finally. This paper is organized as follows. A brief review of mathematical background on the interval analysis is given in the Appendix. In the Section 2, the epsilon-algorithm for eigensolution reanalysis of structures with fairly large changes of parameters is discussed [15]. The Section 3 presents the definition of the interval eigenvalue problems of the interval parameter structures. In the Section 4, the proposed method for evaluating the natural frequencies of structures with fairly large uncertainties of parameters is developed. In Section 5, two engineering examples are given to illustrate the application of the proposed method. The results obtained by the proposed method are compared with those obtained by the exact solutions and the first-order approximation. The conclusions are drawn in Section 6.

2. Eigensolution reanalysis with the epsilon-algorithm

2.1. The epsilon-algorithm

The epsilon-algorithm was presented [16–18] to accelerate the convergence of a sequence.

Given a vector sequence $\{\mathbf{s}_0, \mathbf{s}_1, \mathbf{s}_2, \ldots\}$, we construct the iterative form to obtain the vector sequence in the epsilon-algorithm as follows:

$$\boldsymbol{\varepsilon}_{-1}^{(j)} = 0 \tag{1}$$

$$\boldsymbol{\varepsilon}_{0}^{(j)} = \mathbf{s}_j \tag{2}$$

$$\boldsymbol{\varepsilon}_{k+1}^{(j)} = \boldsymbol{\varepsilon}_{k-1}^{(j+1)} + \left[\boldsymbol{\varepsilon}_{k}^{(j+1)} - \boldsymbol{\varepsilon}_{k}^{(j)}\right]^{-1} \quad j,k = 0,1,2,\ldots \tag{3}$$

The iterative formulae (1)–(3) are similar to the scalar case except that it requires the inverse of a vector. The definition of the inverse of the vector is given by Roberts [19]

$$\mathbf{u}^{-1} = \frac{\mathbf{u}^*}{(\mathbf{u}^H\mathbf{u})} = \frac{\mathbf{u}^*}{\sum_{i=1}^{n}|u_i|^2} \tag{4}$$

where the $\mathbf{u}^*$ denotes the conjugate of $\mathbf{u}$ and $\mathbf{u}^H$ the conjugate and transpose of $\mathbf{u}$. It should be noted that for the undamped structure the modal vectors are real. Thus, $\mathbf{u}^*$ in Eq. (4) should be replaced by $\mathbf{u}$, and $\mathbf{u}^H$ should be replaced by $\mathbf{u}^T$. In this case, $\mathbf{u}^{-1}$ in Eq. (4) is just a normalized version of $\mathbf{u}$ itself. The vector epsilon-algorithm table can be constructed by Eqs. (1)–(3).

2.2. The epsilon-algorithm for accelerating the convergence of Neumann series

The eigenproblem for the structure is as follows

$$\mathbf{K}_0\mathbf{u}_0 = \lambda_0\mathbf{M}_0\mathbf{u}_0 \tag{5}$$

where $\mathbf{K}_0$ and $\mathbf{M}_0$ are the stiffness and mass matrices of the finite element assemblage. This equation will be referred to as the initial problem in the following discussions. The perturbation method studies the changes of eigenvalues of the system subjected to changes in its design parameters. Therefore, if the initial system is represented by Eq. (5), the problem becomes that to determine $\mathbf{u}$ and λ when $\mathbf{K}_0$ and $\mathbf{M}_0$ are perturbed to the form $\mathbf{K}_0 + \Delta\mathbf{K}$ and $\mathbf{M}_0 + \Delta\mathbf{M}$, respectively.

$$\mathbf{K}\mathbf{u} = \lambda\mathbf{M}\mathbf{u} \tag{6}$$

where

$$\mathbf{K} = \mathbf{K}_0 + \Delta\mathbf{K} \tag{7}$$

$$\mathbf{M} = \mathbf{M}_0 + \Delta\mathbf{M} \tag{8}$$

in Eqs. (6)–(8), if we introduce the following notations

$$\mathbf{f}_0 = \lambda_0\mathbf{M}_0\mathbf{u}_0 \tag{9}$$

$$\mathbf{f} = \lambda\mathbf{M}\mathbf{u} \tag{10}$$

$$\Delta\mathbf{f} = \lambda\mathbf{M}\mathbf{u} - \lambda_0\mathbf{M}_0\mathbf{u}_0 \tag{11}$$

then we obtain

$$(\mathbf{K_0} + \Delta\mathbf{K})\mathbf{u} = \mathbf{f_0} + \Delta\mathbf{f}. \tag{12}$$

It follows that

$$\begin{aligned}\mathbf{u} &= (\mathbf{K}_0 + \Delta\mathbf{K})^{-1}(\mathbf{f}_0 + \Delta\mathbf{f}) \\ &= (\mathbf{I} + \mathbf{K}_0^{-1}\Delta\mathbf{K})^{-1}\mathbf{K}_0^{-1}(\lambda_0\mathbf{M}_0\mathbf{u}_0 + \lambda_0\Delta\mathbf{M}\mathbf{u}_0) \\ &= (\mathbf{I} + \mathbf{B})^{-1}\mathbf{K}_0^{-1}(\lambda_0\mathbf{M}_0\mathbf{u}_0 + \lambda_0\Delta\mathbf{M}\mathbf{u}_0)\end{aligned} \tag{13}$$

In Eq. (13), $\mathbf{B} = \mathbf{K}_0^{-1}\Delta\mathbf{K}$, λ and $\mathbf{u}$ were approximated by λ_0 and $\mathbf{u}_0$, respectively.

By using the Neumann series expansion, we have

$$\tilde{\mathbf{u}} \approx (\mathbf{I} - \mathbf{B} + \mathbf{B}^2 - \cdots)\mathbf{K}_0^{-1}(\lambda_0\mathbf{M}_0\mathbf{u}_0 + \lambda_0\Delta\mathbf{M}\mathbf{u}_0) \tag{14}$$

then obtaining a series

$$\begin{aligned}\tilde{\mathbf{u}}_0 &= \mathbf{K}_0^{-1}(\lambda_0\mathbf{M}_0\mathbf{u}_0 + \lambda_0\Delta\mathbf{M}\mathbf{u}_0) \\ \tilde{\mathbf{u}}_1 &= -\mathbf{K}_0^{-1}\Delta\mathbf{K}\mathbf{K}_0^{-1}(\lambda_0\mathbf{M}_0\mathbf{u}_0 + \lambda_0\Delta\mathbf{M}\mathbf{u}_0) = -\mathbf{K}_0^{-1}\Delta\mathbf{K}\tilde{\mathbf{u}}_0 \\ \tilde{\mathbf{u}}_2 &= -\mathbf{K}_0^{-1}\Delta\mathbf{K}\tilde{\mathbf{u}}_1 \\ &\vdots \\ \tilde{\mathbf{u}}_s &= -\mathbf{K}_0^{-1}\Delta\mathbf{K}\tilde{\mathbf{u}}_{s-1} \quad s = 3,4,\ldots\end{aligned} \tag{15}$$

Assume the solution of the Eq. (6) has the following form

$$\tilde{\mathbf{u}} = \tilde{\mathbf{u}}_0 + \tilde{\mathbf{u}}_1 + \cdots + \tilde{\mathbf{u}}_s + \cdots \tag{16}$$

We define a vector sequence $\{\mathbf{s}_0, \mathbf{s}_1, \ldots, \mathbf{s}_s, \ldots\}$ where $\mathbf{s}_0 = \tilde{\mathbf{u}}_0$, $\mathbf{s}_1 = \tilde{\mathbf{u}}_0 + \tilde{\mathbf{u}}_1$, $\mathbf{s}_2 = \tilde{\mathbf{u}}_0 + \tilde{\mathbf{u}}_1 + \tilde{\mathbf{u}}_2$, and in general

$$\mathbf{s}_i = \sum_{j=0}^{i}\tilde{\mathbf{u}}_j \quad i = 0,1,2,\ldots,s,\ldots \tag{17}$$

To accelerate the convergence of the sequence (17), using the vector epsilon-algorithm (Eqs. (1)–(3)), the sequence, $\{\mathbf{s}_0, \mathbf{s}_1, \ldots, \mathbf{s}_s, \ldots\}$, can be obtained. Refs. [16–18] have pointed out that in the epsilon iterative formulae, when i is odd, the $\boldsymbol{\varepsilon}_i^{(j)}$ are meaningless, and that when the $i = 2k \quad (k = 1,2,\ldots)$, the vector $\boldsymbol{\varepsilon}_{2k}^{(j)}$ is the Shanks transformation. The solution is the last even row in the epsilon-algorithm,

$$\mathbf{u} = \boldsymbol{\varepsilon}_{2k}^{(j)} \tag{18}$$

Compute the eigenvalues using the Rayleigh quotients

$$\lambda_i = \frac{\mathbf{u}_i^T\mathbf{K}\mathbf{u}_i}{\mathbf{u}_i^T\mathbf{M}\mathbf{u}_i} \tag{19}$$