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Reserve prices in all-pay auctions with complete information ☆

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ABSTRACT

We introduce reserve prices in the literature concerning all-pay auctions with complete information, and reconsider the case for the so-called Exclusion Principle (namely, the fact that the seller may find it in her best interest to exclude the bidders with the largest willingness to pay for the prize). We show that a version of it extends to our setting. However, we also show that the Exclusion Principle: (a) does not apply if the reserve price is large enough; (b) does not extend if the seller regards bidders' valuations as identically independently distributed according to a monotonic hazard rate. Preliminary results for the case of independent ex-ante asymmetric bidders suggest that the case for it in settings with positive reserve prices is actually tenuous.

1. Introduction

Auction models are prototypes of competitive settings, and they are used in several branches of the economic literature. In particular, the so-called (first-price) all-pay auction is used (among others) by Hillman and Riley (1989), Baye et al. (1993) and Che and Gale (1998) to model the lobbying process. This type of auction fits the lobbying game well, since a lobbyist's contribution is not typically returned if his efforts are unsuccessful,[1] and indeed this literature has elaborated a number of interesting results. In particular, Hillman and Riley (1989) prove that, if there is some asymmetry among bidders/lobbyists, the politically contestable rent is not totally dissipated even in the case of a large number of potential contenders. In addition, Baye et al. (1993) show that a seller/politician wishing to maximize her revenue may find it in her best interest to exclude certain lobbyists from the "finalist" short list (the so-called "Exclusion Principle"), particularly those lobbyists valuing the political prize most (in order to raise incentives to spend for the likely losers). Che and Gale (1998) show a somehow related result: namely, the imposition of an exogenous cap on individual lobbying contributions may have the adverse effect of increasing total expenditure (by increasing competition among lobbyists).

It has to be stressed that the quoted literature refers to the case of *complete* information (according to standard terminology: see e.g. Mas-Colell and Whinston, 1995: section 23, Appendix B): this means that, at the time of bidding, any detail of the setting is common knowledge to all the bidders, including others' evaluations of the prize. In particular, the

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[1] This feature is also shared by other economic and social games, such as patent races and sports.

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working of the Exclusion Principle also requires that bidders' evaluations are known to the seller (at the time exclusion is decided), a rather unusual assumption in auction theory (in fact, a general rationale for using an auction mechanism is exactly the fact that how much bidders value the prize to be allocated among them is their private knowledge). In addition, it has been noticed by Gale and Stegeman (1994) that the Exclusion Principle depends on the assumption that the politician must award the prize (i.e., it cannot withhold it nor use take-it-or-leave-it offers).[2] This assumption can be justified in the lobbying setting if the politician is unable to refuse credibly to allocate the political rent: e.g., Baye et al. (1993) refer to the choice of a city to host the Olympic Games.

In this paper we discuss the case in which the seller can possibly use a different exclusion tool, namely a reserve price (a common mechanism in auction theory),[3] which does not require the seller to know bidders' evaluations. After characterizing the equilibrium of the all-pay auction with an exogenously given reservation price, we show that the seller would indeed prefer a strictly positive reserve price, which also increases the overall outcome efficiency (it might decrease the efficacy of the lobbying process through higher rent dissipation). We show that a generalized version of the Exclusion Principle holds: namely, that a seller lacking the full-fledged bargaining ability to make a take-it-or-leave-it offer to the highest-evaluation bidder (the intuitive optimal mechanism for the seller) could still find optimal to exclude some bidders from her short list even when using a positive reserve price. However, the case for the Exclusion principle becomes weaker in such a setting, since it cannot apply if the reserve price is high enough.

We then discuss a setting in which, in an all-pay auction with complete information among the bidders, the seller is not fully informed while setting her reserve price and/or considering some exclusion. Namely, we consider the case the seller ex ante regards bidders' "ad-interim" valuations as unknown realizations of random variables.[4] In such a setting Menicucci (2006) strikingly shows that, even if the seller regards the bidders' private valuations as identically and independently distributed (iid), for some information structures excluding all but two bidders (randomly selected) increases the seller's expected revenue (yet another version of the Exclusion Principle). We characterize the optimal reserve price in such a setting and extend the result in Bertoletti (2008): namely, while using a positive reserve price, the seller wishes no exclusion if she regards bidders' valuations as iid distributed according to a monotonic hazard rate (a feature of many common distributions). Preliminary results for the case of independent but ex-ante asymmetric valuations seem to suggest that the case for the Exclusion Principle in settings with positive reserve prices is indeed tenuous.

2. The all-pay auction with complete information and a reserve price

Consider the following setting: n (risk-neutral) agents (the "buyers") bid for a prize (there is no resale possibility). Bidder i's (private) valuation of the prize is v_i ($i = 1, \ldots, n$), and we order the bidders in such a way that $v_1 > v_2 > \ldots > v_{n-1} > v_n > 0$.[5] The "rules" of the auction can include a reserve (minimum) price $p_r \geq 0$, i.e., a price below which the prize is not assigned. In particular, let us indicate with b_i the bid of agent i. In an (first-price) all-pay auction, bidder i receives the prize if $b_i > \text{Max}\{b_{j\neq i}\}$ and $b_i \geq p_r$, and in that case his payoff is $v_i - b_i$, whereas his payoff is $-b_i$ if he loses (ties are broken randomly). Assuming $p_r = 0$, Hillman and Riley (1989) and Baye et al. (1993, 1996) show that in the unique Nash equilibrium agent 1 uses the uniform distribution $F_1(b_1) = b_1/v_2$ on the support $[0,v_2]$, while agent 2 uses $F_2(b_2) = 1 - v_2/v_1 + b_2/v_1$ on the same support (note that this amounts to the fact that agent 2 randomises between $b_2 = 0$ and the uniform distribution on $[0,v_2]$ with probabilities respectively $1 - v_2/v_1$ and v_2/v_1). Agents $j = 3, \ldots, n$ bid $b_j = 0$ with probability 1. The prize is then given to agent 1 with probability $1 - v_2/(2v_1) > ½$ and to agent 2 with probability $v_2/(2v_1) < ½$ (note that in the latter event the result is not ex-post efficient, and thus it would not be stable in the case of a resale opportunity). Agent 1 receives a (expected) payoff of $U_1(v_1, v_2) = v_1 - v_2$, while the (expected) payoffs of the other agents are zero; i.e., $U_j(v_1, v_2) = 0, j = 2, \ldots, n$. The expected total payment to the seller is $p(v_1, v_2) = p_1(v_1, v_2) + p_2(v_1, v_2) = v_2/2 + (v_2/v_1)(v_2/2) = v_2\,(1 + v_2/v_1)/2 < v_2$, where p_i is the expected payment of agent $i = 1,2$.

The previous results show that the outcome of an all-pay auction is not ex-ante efficient, since for the expected social welfare W the following inequalities hold $v_2 < W(v_1, v_2) = v_1 - v_2 + p(v_1, v_2) < v_1$.[6] From the perspective of the economic theory of lobbying, they illustrate the possibility that, even if the number of potential contenders is large, asymmetries among players might imply that the political rent is not fully dissipated (see Hillman and Riley, 1989: pp. 18–19). In addition, note that $\partial p/\partial v_1 < 0$ and $\partial p/\partial v_2 > 0$ ($p(\cdot)$ can be proved to be convex): indeed, Baye et al., (1993) show that a politician (the seller in the auction) wishing to maximize her revenue should be willing to select the two active lobbyists (the bidders) i^* and i^*+1 in order to maximize $p(v_i, v_{i+1})$. This implies that she might find it in her best interest to exclude lobbyists from 1 to i^*-1 from her "finalists short list", if she is allowed to (there is no point in excluding bidders from i^*+2 to n). This could

[2] Gale and Stegeman (1994) discuss a mechanism in which the seller need not to award the prize to the highest bidder, and prove that it delivers to the seller an higher revenue.

[3] In a lobbying game a positive "reserve price" could perhaps be interpreted as the politician's ability to postpone (possibly *sine die*) the final decision.

[4] Actually, this is the standard assumption in a "complete information" setting: again see Mas-Colell et al. (1995).

[5] The possibility of ties in the valuations is ignored here. This can be justified by assuming that the v_i are ex-ante continuously independently distributed, so that case has *a priori* a zero probability (in an all-pay auction ties may imply the existence of multiple Nash equilibria which are not necessarily revenue equivalent: see Baye *et alii*, 1996 and footnote 9 below).

[6] For the sake of simplicity, we assume that the seller's evaluation of the prize is zero.