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Stacking sequence design of a composite wing under a random gust using a genetic algorithm

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Abstract

The layup optimization by genetic algorithm (GA) for the composite wing subject to random gust is presented. The aim of optimization is to maximize the strength of wing and the failure index of Tsai–Hill criterion is used as the objective function. The failure index is calculated by Monte Carlo simulation because the external loading and the material properties have random characteristics. The optimization results are validated by comparing the failure probability of the initial and optimal designs. In addition, the optimum by maximum stiffness criterion is also obtained to show that current objective function is appropriate for the design of composite wing.

Keywords: Random gust; Probability of exceedance; Monte Carlo simulation; Stacking sequence; Genetic algorithm

1. Introduction

The use of composite materials in aerospace vehicles can result in a significant increase in payload, weight reduction and fuel efficiency. For example, advanced composite materials are widely used for modern wing designs to reduce the structural weight. Thus research efforts have been devoted to the optimal design of wing structures in connection with various objectives and constraints [1–5]. But, to author's knowledge, it can be seen that not much work has been reported on the probabilistic wing design and analysis. Penmetsa and Grandhi [6] calculated the failure probability using interval analysis where the wing skin thickness and the loading were considered to be available as an interval. Mahadevan and Liu [7] developed the system reliability analysis procedure and applied it to the analysis of composite wing structure. In their work, material properties, ply thicknesses and orientations, and pressure loads were assumed to be random variables.

In this paper, the stacking sequence of composite wing subject to random gust loading is optimized to have maximum strength. By the atmospheric turbulence during flight, the wing may experience the excessive structural loading and vibration which cause structural failure. Thus, the failure index at the wing root is selected as the objective function of the current optimization problem. In the analysis, gust loading is represented by the induced wing root bending moment which also has probabilistic characteristics. So, the probabilistic approach is needed for the calculation of failure index and Monte Carlo simulation is used in this research.

To evaluate the fitness function via Monte Carlo simulation, first, the probabilistic model for random variables should be defined. The probabilistic distribution of induced moment can be obtained by applying the concept of frequency of exceedance to the results of power spectral density analysis. The material properties of composite wing are always subject to a certain amount of uncertainty and are assumed to show normal distribution in this paper. With the random variables for the bending moment and material properties, the maximum failure index at the wing root can be found by Monte Carlo simulation.

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As next stage, the maximum failure index is minimized by using the ply angles as design variables. In practical applications, ply angles are limited to a fixed set of angles such as 0°, ±45°, 90°. Thus, combinatorial optimization methodology is needed for stacking sequence optimization to handle discrete ply angles. It is known that genetic algorithm (GA) is one of the suitable choices of the methodology for discretized optimization problems. GA is based on the mechanics of natural selection, crossover and mutation, and searches the optimal solution through random probability methods without auxiliary information such as derivatives or intelligently chosen starting points. GA has been successfully used in composite structural design. Riche and Haftka [8] proposed genetic algorithm to optimize the stacking sequence of composite laminate for buckling load maximization. For the same problem, Liu et al. [9] has provided permutation genetic algorithm. A recessive gene repair strategy was introduced by Todoroki and Haftka [10] for satisfaction of the given constraints. In the present study, GA with a repair strategy is adopted for the optimization of layup design. The balanced symmetric layup constraint and limitation of four contiguous layers are implemented by the repair strategy.

The optimal solutions obtained by failure criterion are verified by two ways. The failure probabilities are obtained for the initial and optimal designs and the improvement of structural safety is evaluated. Also, the optimal solutions by maximum stiffness criterion are obtained to confirm the appropriateness of the current objective function.

2. Statistical model of gust loading

2.1. Gust response analysis

By considering the atmospheric turbulence as a stationary Gaussian random process, the power spectral methods can be used for finding the root mean square (RMS) values of the various outputs, such as bending moments, accelerations, and deflections, induced by random gust. It is known that if $W_G(\tau)$ represents the vertical gust velocity and $\psi(\tau)$ denotes a system response, such as bending moments, then the power spectrum of the response is given by

$$\Phi_{\psi}(\omega) = |H(\omega)|^2 \Phi_{W_G}(\omega) \tag{1}$$

where $H(\omega)$ is the frequency response function and the system is assumed to be linear. The power spectrum of the response can be used to determine the RMS values of the response σ_{ψ} as

$$\sigma_{\psi}^2 = \int_{-\infty}^{\infty} \Phi_{\psi}(\omega)\,\mathrm{d}\omega \tag{2}$$

For the power spectral density (PSD) of gust velocity, the von Kármán model [6], which is defined by Eq. (3), is used in this paper

$$\Phi_{W_G}(\omega) = \frac{\sigma_{W_G}^2 \tau_g}{\pi} \frac{1 + (8/3)[1.339\tau_g\omega]^2}{[1 + (1.339\tau_g\omega)^2]^{11/6}} \tag{3}$$

where $\tau_g = L_g/V$, L_g is the scale of turbulence and V is flight velocity, respectively. Also, RMS value of the gust velocity is given by

$$\sigma_{W_G}^2 = \int_{-\infty}^{\infty} \Phi_{W_G}(\omega)\,\mathrm{d}\omega \tag{4}$$

For example, Fig. 1 shows a log–log plot of $\Phi_{W_G}(\omega)$ as a function of ω, where the quantity $\sigma_{W_G}^2$ and the ratio τ_g were chosen to be unity.

2.2. Probability of exceedance

If the PSD of bending moment was calculated by power spectral analysis, the probability distribution of the moment can be obtained using the concept of frequency of exceedance. For any load quantity, the frequency of exceedance, $N(y)$, is defined as the number of crossings of a given level y per unit time. The following empirical formula is used for calculating the $N(y)$ in gust response analysis [11]:

$$N(y) = N_0\left[P_1 \exp\left(-\frac{y/\overline{A}}{b_1}\right) + P_2 \exp\left(-\frac{y/\overline{A}}{b_2}\right)\right] \tag{5}$$

In Eq. (5), N_0 is the number of crossing rate of level 0 and $\overline{A}$ is the ratio of the RMS of output to that of the gust, which are given by

$$\overline{A}^2 = \frac{\int_0^{\infty} \Phi_{\psi}(f)\,\mathrm{d}f}{\int_0^{\infty} \Phi_{W_G}(f)\,\mathrm{d}f} \quad \text{and} \quad N_0^2 = \frac{\int_0^{\infty} f^2 \Phi_{\psi}(f)\,\mathrm{d}f}{\int_0^{\infty} \Phi_{\psi}(f)\,\mathrm{d}f} \tag{6}$$

In practice, the integrals defining $\overline{A}$ and N_0 are evaluated only up to a reasonable upper limit. The values for parameters, P_1, P_2, b_1 and b_2 can be determined from FAA regulations and depend only on altitude.

Once $N(y)$ in Eq. (5) is obtained, it is easy to calculate the probability of exceedance. In this paper, the following equation is used for relating frequency of exceedance and probability [11]:

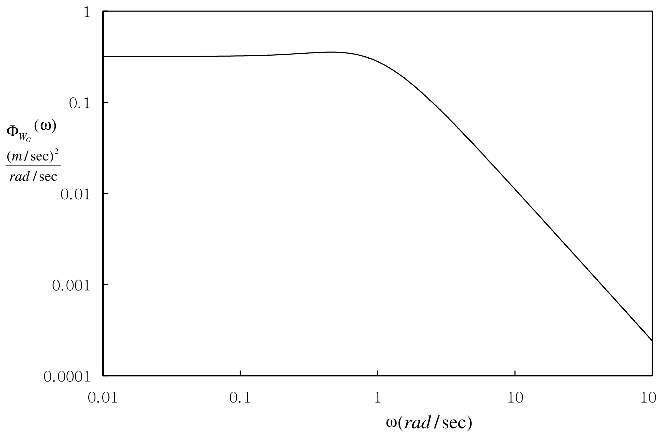

Fig. 1. Von Kármán PSD function.