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FEM formulation of homogenization method for effective properties of periodic heterogeneous beam and size effect of basic cell in thickness direction

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ABSTRACT

This paper applies the asymptotic homogenization method to predict effective properties of periodic heterogeneous beam structures. A improved FEM formulation and algorithm of the unit cell problems is developed. The effective properties are rewritten in terms of the nodal quantities of the FEM model of the unit cell. With this approach, periodic beam structure of complicated microstructure can be modeled by various elements and modeling techniques and solved by using the commercial software as a black box. Numerical examples illustrate the versatility and efficiency of the method. Finally, the size effect of basic cell in thickness direction is studied.

1. Introduction

With the development of industrial technology, the configuration of engineering structure is becoming more and more complicated to meet the requirement of multifunctional, lightweight and efficient performance. Slender heterogeneous beam structures with the cross sectional dimensions being significantly smaller than their length along the axial direction are widely used in engineering. Using conventional numerical methods to analyze the overall behavior of these structures may be tedious and leads to heavy computations. Since that, this kind of structures is often reduced as a homogeneous 1-D beam, like an Euler–Bernoulli or Timoshenko beam.

Various approaches were developed for dimensional reduction of these heterogeneous beam structures. The behavior of stochastically heterogeneous beams including cross sectional as well as longitudinal heterogeneity was studied in [1] based on statistical characteristics of average displacements, reaction forces and their statistical variance. Corn et al. [2] presented a condensation method for simplifying finite element models of structures having a beam-like global dynamical behavior. Wang and Cheng [3,4] developed a reduced beam model for dynamic analysis of the heterogeneous beam structure using the modified physical assumption such as block-wise rigid body motion and super beam interpolation. Carrera et al. [5–7] constructed higher-order beam models by adopting various expansions (polynomial expansion, Taylor expansion, Lagrange expansion) of the displacement field in the cross section of beam structure in the framework of the Carrera Unified Formulation (CUF). Kennedy and Martins [8,9] presented a homogenization-based theory for layered orthotropic beams and anisotropic beams with accurate through-section stress and strain prediction, which builds a kinematic description of the beam from a linear combination of fundamental state solutions.

Heterogeneous beam structures consisting of unit cells arranged periodically in its longitudinal direction are one important class of slender heterogeneous beam structures. Sandwich beams, ribbed boxes and stranded ropes are such structures. In order to simplify them, one key step is to obtain the equivalent macro-mechanical properties based on the micro-structure of unit cell. Once the equivalent properties are available, the original heterogeneous structure is approximated by a homogeneous Euler beam, by which its global behavior such as static deformation and low order natural vibration frequencies can be predicted within engineering accuracy.

The variational asymptotic method (VAM) developed by Cesnik and Hodges [10] was one of the most successful approaches in dealing with arbitrary sectional properties, but limited to the interior solutions in constant-section beams. Yu et al. [11,12] and Lee et al. [13] extended VAM and reduced the original 3-D problem

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to one dimensional beam through simultaneous homogenization and dimensional reduction. Commercial code VABS was developed to implement this method numerically, but it is not apparent in the original formulation how to define an adequate set of boundary conditions, implement it numerically or adapt it to conventional engineering models.

Various methods have been developed to estimate the effective properties of composite continuum, such as self-consistent scheme (SCS) [14], generalized self-consistent scheme (GSCS)[15], the Mori–Tanaka method (M–T) [16], representative volume element method (RVE) and asymptotic homogenization (AH) method. The theoretical approaches SCS, GSCS and M-T can provide very simple and closed-form expressions for composite continuum with single inclusion in matrix and approximate estimation for more complicate composite continuum [17,18]. RVE method and AH method are two widely used numerical methods for composite with complicated microstructures. The RVE method has clear mechanical conception and implements simply, but it is not based on rigorous mathematical theory and only provides approximate estimation. AH method is regarded as a powerful two-scale method, since it is based on rigorous mathematical perturbation theory and can be applied to microstructure of general shape and arbitrary heterogeneity. It has been successfully used in predicting effective modulus of 3-D and 2-D periodic materials both analytically and numerically (see [19–21]). The AH method was also applied to evaluate the vibration modes of large repetitive structures by Daya and Potier-Ferry [22–24]. Kolpakov and Kalamkarov [25–27] developed the asymptotic homogenization theory for heterogeneous beam structures with periodicity along its longitudinal axis and extended the application scope of AH method. Through elaborate and complicated analytical derivation, the initial 3-D heterogeneous problem splits into a microscopic 3-D problem posed on the unit cell of the structure and a macroscopic beam problem. Based on this theory, they analyzed a group of beam structures and some bounds of the effective stiffness were obtained using variational principles in Kolpakov [26]. However, analytical solutions are difficult to obtain for complex microstructures and approximate solutions are substituted with some simplifications. The numerical approach to calculate the effective properties is not given any priority in their work, because their formulation is hard to combine with the finite element method.

Formal asymptotic method (FAM) as a modification to the asymptotic homogenization method has also been used to study the case of periodic beam structures (Buannic and Cartraud [28,29]; Kim and Wang[30]), which is a direct application of the two-scale method in the original 3-D governing equations of beam structures to perform an asymptotic homogenization. But it is not easy to relate the equations derived in FAM method with simple engineering models and is difficult to implement numerically either.

Cartraud and Messager [31] followed Kolpakov [25,27], Kalamkarov and Kolpakov [32] and Buannic and Cartraud [28,29] and applied the asymptotic homogenization theory. The resulting basic cell problem was implemented in a commercial finite-element package (Samcef), but the solution procedure is not written in the framework of FEM. Furthermore, the unit cells in their numerical examples are all modeled by three dimensional solid elements, which can be inefficient in many cases. One of recent important development by Dizy et al. [33] introduced a general methodology to evaluate both the elastic constants and local buckling characteristics of composite beams with spanwise periodic properties. They presented a comprehensive review of the topics and pointed out that the existing solutions either were limited to constant-section geometries, required intricate implementations, or user-created modules or subroutines in a standard finite-element solution package. Their method was implemented into a off-the-shelf finite-element solver (Abaqus), and takes advantage of some modeling features of the commercial packages such as the tie constraints to simplify model generations. However, in their paper the FEM formulation was not elaborated, and ten combinations of loading cases were needed to calculate the effective stiffness. All numerical examples are modeled by 3D solid elements.

A new implementation of the asymptotic homogenization (NIAH) method has been developed by Cheng et al. [34] to predict effective properties of periodic materials with periodicity in two and three dimensions, and has been extended to the homogenization method for plate structures with periodicity in-plane by Cai et al. [35]. The new implementation has a rigorous mathematical foundation of the asymptotic homogenization method, and can be simply implemented by using commercial software as a black box. All kinds of elements and modeling techniques available in commercial software can be used to discretize the unit cell, so the complicated unit cell model may remain a small scale.

Size effect of basic cell is an interesting topic when the homogenization method is applied to practical structure composed of a finite number of basic cells since the basic assumption of homogenization method is that the periodic basic cells extend to infinite. For two and three dimensional continuum, discussion can be seen in many papers (see [36–38]). Here we prefer the term of basic cell to unit cell because it has finite cell number in practical structure.

For periodic heterogeneous beam, the periodicity in the thickness direction does not exist. Since an easy implementation of the rigorous homogenization method for effective properties of periodic heterogeneous beam such as composite beam is not publically available, a common approximate approach to obtain its effective properties consists of two steps. The first step is to obtain the effective properties of the beam material by the 3D homogenization method, which assume infinite periodicity in thickness direction and neglect the effect of free upper, bottom and side surfaces. The second step assumes the beam to be composed of homogenized material and obtains its effective properties by the ordinary beam theory. The accuracy of this common approach is mentioned in many papers. For example, Rpstam-Abadi et al. [39] applied the weighting procedure to the material constants through thickness direction in which the periodic condition for the standard homogenization method is not assumed. Nasution et al. [40] in their work assumed that the periodicity in the thickness direction should be relieved. Size effects of the basic cell for sandwich beams were studied by Dai and Zhang [41], and they found that the traditional AH theory had limitations in capturing size effects of the beams when the scale factor varies.

The present paper develops a finite element formulation for the asymptotic homogenization theory of periodic beam structures, thus extends the NIAH method in [34,35] to the beam structures with complicated microstructures. Here “complicated microstructures” refers to the fact that the micro-structure of the unit cell can be very complicated, for example, the unit cell can be a structure consisting of bars, beams, plates and shells, 2D or 3D continuum, even of multiphase material. Box beams with periodic transverse reinforcement, corrugate beams, sandwich beams with various cores and stranded cables are a few representative examples to be solved in this study. Using the method, the heterogeneous beam structure is reduced to a classical Euler–Bernoulli beam and a 4×4 stiffness matrix is obtained by solving the unit cell problems under four generalized unit strain cases, including extension, twisting, and bending in two directions. The improved formulation can be implemented by following the flowchart in Section 3.2 and using the commercial software as a black box. Arbitrary shape and heterogeneity of the unit cell with various structural components (solid, plate and shell, beam, etc.) and various modeling techniques can be considered. Several examples are carried out to illustrate the validity of the proposed method,