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A Wave Based approach for the dynamic bending analysis of Kirchhoff plates under distributed deterministic and random excitation

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ABSTRACT

For the harmonic analysis of plate bending problems, the Finite Element Method (FEM) is a commonly applied numerical technique. Its element concept with polynomial approximation functions, however, limits its applicable frequency range because of a strongly increasing computational cost. The Wave Based Method (WBM) has can relax this by using wave functions, which satisfy the governing differential equations.

This paper derives two distinct particular solution sets for distributed loads in the WBM. Two numerical validations show the improved efficiency as compared to the FEM. The novel approach is also applied to a plate under a TBL excitation.

1. Introduction

The Finite Element Method (FEM) [1] is one of the most commonly applied simulation technologies to predict the behaviour of dynamic systems since it can tackle geometrically complex problems by dividing the problem domain in small elements. Simple polynomial approximation functions are most commonly used within the elements to describe the dynamic field variables. This procedure, involving small elements with polynomial functions is the strength of the FEM, but it also constitutes a limitation. As the frequency increases, the number of elements required to control the interpolation and pollution errors increases more than linearly [2–4]. The increasing model size creates an upper frequency limit above which the computational cost becomes prohibitive. The FEM should thus be considered as a low-frequency technique.

Significant research is performed in order to alleviate these limitations of the FEM. Important to mention are the so-called meshless methods, where the very fine element discretisation is no longer made. Instead, approximation functions with a higher degree of continuity are applied. The application to thin plates can be found in many fields of dynamic analysis: e.g. nonlinear dynamic fracture and crack growth [5,6], buckling analysis [7,8] and steady-state linear vibration analysis [9,10]. Another class of techniques, partly intersecting the class of the meshless methods, is the group of Trefftz-based methods [11,12]. These methods all apply the same principle; the solution is approximated through a set of so-called Trefftz functions, which inherently satisfy the governing differential equation(s) *a priori* and which may violate conditions at the domain boundary. The best known Trefftz-based methods are the Discontinuous Galerkin Method [13], the Hybrid Trefftz FEM [14], the Method of Fundamental Solutions [15,16], the Variational Theory of Complex Rays [9,17], the Ultra-Weak Variational Formulation [18–20] and the Wave Based Method (WBM) [21], which is the focus of this paper. The key differentiator between these methods is the way in which the boundary and interface conditions are imposed and the specific selection of type of basis functions.

This paper focuses on the Wave Based Method (WBM) [21], which has the potential to alleviate the FEM's frequency limitations for problems of moderate geometrical complexity. As compared to FE models, WB models are much smaller and have a higher convergence rate, which enables faster calculations for the same accuracy or allows going to higher frequencies for the same computational cost. So far, the method has been successfully applied to acoustic problems [22–24], in-plane membrane [25] and plate bending [10] problems, poro-elastic problems [26,27] and, in a hybrid sense with the FEM, to fully coupled vibro-acoustic problems with structural [28] and poro-elastic [29] components.

The WBM for plate bending problems forms the starting point of this paper. The research on these problem settings was initiated

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by Desmet [21]. Vanmaele et al. [10,30] studied this topic more profoundly. So far, however, the WBM only allows for excitation by a prescribed boundary condition, or excitation by a point force inside the domain. Nevertheless, distributed loads are omnipresent in engineering practise. They can cover a wide range of load profiles, ranging from loaded patches to full surface distributed loads. In the former case, concentrated loads are distributed over a small but finite area. Depending on the ratio between the patch dimensions and the governing wavelength, the assumption of a localised point force no longer holds. Examples of the latter case are the vibro-acoustic coupled problems and the excitation by a broadband random excitation, such as e.g. a diffuse acoustic field or a Turbulent Boundary Layer (TBL). By virtue of their stochastic nature, these random excitations are only known in statistical terms. They can however be elegantly described in the wavenumber-frequency domain [31]. Whereas the diffuse field models yield accurate results using only a simple expression, the excitation by a TBL is less trivial to model. Corcos [32] developed an empirical model describing the spectrum of the Turbulent Boundary Layer wall pressure. Over the years, improvements have been made, among others by Efimtsov [33] and Chase [34]. Nevertheless the model still stands as a good estimate of the wall pressure fluctuations' so-called convective ridge [35]. An extensive overview of the modelling of Turbulent Boundary Layer spectra can be found in [36].

The topic of distributed loads in the WBM, however, has only briefly been touched on so far. Desmet [21] used the acoustic wave functions as particular solutions to the plate bending problem in order to have fully coupled vibro-acoustic WB models. Jegorovs [37] introduced the use of the Fourier transform for the derivation of particular solutions and applied this to the so-called light diffusion approximation to the transport theory. In this paper, the existing framework of the WBM for plate bending problems is extended with particular solutions which can incorporate the effect of distributed loads in dynamic plate bending problems. Two different approaches are presented. The first one is derived from the integration of the particular solution for a point force over the loaded surface. In the second approach, particular solutions are derived based on a decomposition of the distributed load in the wavenumber domain. Both approaches are validated in terms of efficiency and accuracy.

The paper is organised as follows; Section 2 reviews the mathematical formulations of the plate bending problem, with its governing dynamic equation and boundary conditions. The WBM for plate bending problems is discussed in Section 3. The existing framework is extended with particular solutions for distributed loads in Section 4. Section 5 demonstrates the potential of the developed functions with a number of academic numerical validation examples, both on a simple rectangular plate and on a more complicated shape. Section 6 uses the newly developed particular solutions to compute the response of a plate under a TBL excitation. The paper ends with a general conclusion on the presented work.

2. Problem definition

Consider a thin flat plate shown in Fig. 1. The steady-state dynamic behaviour can be described by the Kirchhoff theory [38]. According to this thin plate theory, the steady-state out-of-plane displacements $w_z(\mathbf{r})$, with $\mathbf{r} = (x, y)$, are governed by the following fourth order partial differential equation:

$$\mathbf{r} \in \Omega : \nabla^4 w_z(\mathbf{r}) - \left(k_b^s\right)^4 w_z(\mathbf{r}) = \frac{F_z}{D}\delta(\mathbf{r_F}) - \frac{p(\mathbf{r_d})}{D}, \qquad (1)$$

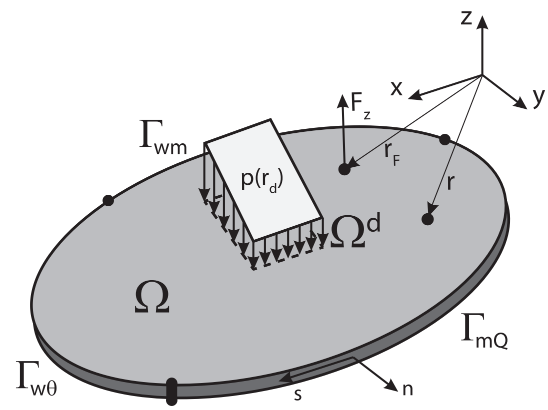

Fig. 1. General plate bending problem.

where $\nabla^4 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}$. The structural wavenumber for plate bending k_b^s and the bending stiffness D are defined as:

$$k_b^s = \sqrt[4]{\frac{\rho t \omega^2}{D}}, \qquad (2)$$

$$D = \frac{E(1 + j\eta)t^3}{12(1 - \nu^2)}, \qquad (3)$$

with t the thickness of the plate, E the elasticity modulus, η the material loss factor, ν the Poisson coefficient, ρ the material density, $\omega = 2\pi f$ the harmonic pulsation and $j^2 = -1$. The plate is excited by a normal point force F_z in the point $\mathbf{r_F} = (x_F, y_F)$ and by a distributed normal load $p(\mathbf{r_d})$ on a part Ω^d of the plate surface Ω.

The Kirchhoff Eq. (1), being a fourth order partial differential equation, requires two boundary conditions at every point on the problem boundary $\Gamma = \partial\Omega = \Gamma_{w\theta} \cup \Gamma_{mQ} \cup \Gamma_{wm}$. For an easy understanding, the boundary conditions which are further used in the paper, are recapitulated. The prescribed values for the out-of-plane displacement, rotation, generalised shear force and bending moment are written as $\overline{w}_z, \overline{\theta}_n, \overline{Q}_n$ and $\overline{m}_n$, respectively. The differential operators associated with these derived quantities, $\mathcal{L}_{\theta_n}, \mathcal{L}_{m_n}$ and $\mathcal{L}_{Q_n}$ are defined as:

$$\mathcal{L}_{\theta_n} = -\frac{\partial}{\partial n}, \qquad (4)$$

$$\mathcal{L}_{m_n} = -D\left(\frac{\partial^2}{\partial n^2} + \nu\frac{\partial^2}{\partial s^2}\right), \qquad (5)$$

$$\mathcal{L}_{Q_n} = -D\frac{\partial}{\partial n}\left[\frac{\partial^2}{\partial n^2} + (2 - \nu)\frac{\partial^2}{\partial s^2}\right], \qquad (6)$$

with n and s the in-plane normal and tangential directions to the plate boundary Γ, as indicated in Fig. 1.

With this notation, the boundary conditions can be expressed as follows:

- Kinematic boundary conditions with prescribed values on displacements and rotations:

$$\mathbf{r} \in \Gamma_{w\theta} : \begin{cases} R_{w_z}(\mathbf{r}) = w_z(\mathbf{r}) - \overline{w}_z(\mathbf{r}) = 0 \\ R_{\theta_n}(\mathbf{r}) = \mathcal{L}_{\theta_n}[w_z(\mathbf{r})] - \overline{\theta}_n(\mathbf{r}) = 0 \end{cases}. \qquad (7)$$

  The clamped edge boundary condition, which is used in the validations, is a kinematic boundary condition with all displacements and rotations constrained.
- Mechanical boundary conditions with prescribed values of the stress resultants. However, since only two boundary conditions can be imposed, the shear force q_n and the twisting moment m_{ns} are combined into a generalised shear force: