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Bayesian model selection of hyperelastic models for simple and pure shear at large deformations

T.G. Ritto [a,*], L.C.S. Nunes [b]

[a] Federal University of Rio de Janeiro, Department of Mechanical Engineering, Rio de Janeiro, Brazil
[b] Universidade Federal Fluminense, Department of Mechanical Engineering (TEM/PGMEC), Laboratory of Opto-Mechanics (LOM/LMTA), Rio de Janeiro, Brazil

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ABSTRACT

Hyperelastic materials are extensively employed in a wide range of applications. Although there are well-established models for describing the mechanical behavior of the hyperelastic materials, relatively few papers have attempted to rank different models. This paper aims to identify parameters of some constitutive models for pure and simple shear of an incompressible isotropic hyperelastic material under large deformations, and also aims to propose a strategy to rank different models. The constitutive models considered in the present analysis are the following: Mooney–Rivlin, Yeoh, Ogden (1 and 2 terms), Lopez-Pamies (1 and 2 terms), and Gent. In the first part of the paper, the Bayesian framework is applied for the identification of the parameters of the models, where experimental data are used to update the prior probabilistic model of the unknown parameters. The Maximum a Posteriori Estimate is obtained, and the error between the model prediction and the experimental data is computed to rank the models. In the second part of the paper, the Bayesian framework is again employed, but now as an strategy for model selection. Instead of ranking the models using the error between the model prediction and the experimental result, more ingredients, such as the Ockham factor, are taken into account for the model selection. The results indicate that all models and experimental results for pure shear are in good agreement, but Mooney–Rivlin, Gent and Yeoh models were not able to well describe the available experimental data from simple shear. The best model for pure shear is Mooney–Rivlin and the best model for simple shear is Ogden (2 terms), considering the available experimental data and the criteria proposed in the present paper.

1. Introduction

We think through models, and we use them all the time in Engineering for design, manufacture and maintenance. However, how good is a model to make accurate predictions? Which model is better to describe a given experiment? There are many different approaches to establish a criterion to answer these questions. In the present paper we rank different models by (1) computing the error between model prediction and experimental data, and (2) through a Bayesian strategy for model selection. Specifically, this work is interested in constitutive models of an incompressible isotropic hyperelastic material under simple and pure shear at large deformations.

Nowadays, hyperelastic materials, such as rubber, are extensively employed in a wide range of applications. Many of these applications involve development of components with complex geometries under a variety of loading conditions. Sometimes uncertainties in large deformation processes must be taken into account [1]. In general, numerical simulation is used to solve these types of problems [2]. In this way, a constitutive model for predicting the mechanical behavior of the material is often required. There are several models to predict the mechanical behavior of hyperelastic materials that can be found in the literature [3–6]. The stress–strain response of these types of materials is derived from a strain-energy function. Here, for comparison purpose, four classical and well-known models were chosen: Mooney–Rivlin [7,8], Yeoh [9], Gent [10] and Ogden [11]. First- and second-order Ogden were taken into account. Besides, a more recent model proposed by Lopez-Pamies [12] (1 and 2 terms) is also considered. These models describe well the stress–strain relationship for a uniaxial tension test. However, few of them are capable of describing complex load conditions. For that reason, in the present paper, simple and pure shear tests were selected [13–16].

* Corresponding author.
E-mail addresses: thiagoritto@gmail.com, tritto@mecanica.ufrj.br (T.G. Ritto).

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Although there are well-established models for describing the mechanical behavior of the hyperelastic materials, relatively few papers have attempted to rank different models. For instance, in [6], a comparison of hyperelastic models for rubberlike materials is made, where twenty hyperelastic models were considered, and the ranking of the models was made by curve fitting.

The large number of hyperelastic material models can pose significant challenges to the experimentalist for selecting the most suitable model, i.e. a robust model with a small number of parameters. To overcome this difficult, a selection process can be used. In this way, the main contribution of the present work is to employ the Bayesian approach to rank different hyperelastic models for pure and simple shear states. The Bayesian framework [17–19] is used to identify the parameters and also to rank the different models. It is important to emphasize that this technique is advantageous because it provides more information than just mean squared error.

The strategy proposed by Beck and co-workers [17,20–23] is employed to identify the parameters of the models and to rank the different models. In other words, the probability models, including the prediction error probability model, are updated with the available experimental data, such that a quantitative assessment of the model accuracy is obtained. For the Bayesian identification, the constitutive model parameters are modeled as random variables, and a prior probability density function (pdf) is assigned for them. An additive Gaussian noise model, with unknown variance, is assumed, and the prior pdf is updated with the experimental data, with the aid of Metropolis Hastings/Markov Chain Monte Carlo (MCMC) technique [24,25]. The Maximum a Posteriori Estimate (MAP) is considered and a confidence region is constructed for the model's prediction. The experimental data obtained by Moreira and Nunes [14] are used for the comparisons.

To rank the constitutive models, first we compute the error between the model prediction and the experimental data. Then, employing a Bayesian model selection strategy, we investigate the probability of each model, given the experimental data. With such strategy, not only the error between the model prediction and the experimental data is taken into account, but also the Ockham factor is considered, penalizing additional parameters of the models. In general, a good constitutive model requires few parameters to describe the experimental data accurately. Models with a large number of parameters can be used to fit complex response, but special care is required in order to avoid over-fitting.

This paper is organized as follows. Next section depicts the models analyzed. Section 3 presents the identification procedure of the model parameters within the Bayesian framework. Section 4 explains the Bayesian model selection strategy and Section 5 presents the results. Finally, the concluding remarks are made in the last Section.

2. Simple and pure shear models

The concepts of simple and pure shear are well known in mechanics. Simple shear is related to a state of deformation, while pure shear denotes a state of stress that is characterized by tr $\sigma = 0$. The Cauchy stress tensor σ of an incompressible isotropic hyperelastic material may be expressed in terms of strain-energy function Ψ [8,26].

- for pure shear

$$\sigma_{11} = 2(\lambda^2 - \lambda^{-2})\left(\frac{\partial\Psi}{\partial I_1} + \frac{\partial\Psi}{\partial I_2}\right)_{\mathbf{ps}}, \tag{1}$$

with the strain invariants denoted by $I_1 = I_2 = \lambda^2 + \lambda^{-2} + 1$.

- for simple shear

$$\sigma_{12} = 2k\left(\frac{\partial\Psi}{\partial I_1} + \frac{\partial\Psi}{\partial I_2}\right)_{\mathbf{ss}}, \tag{2}$$

where k is defined as amount of shear. In this case, the amount of shear can be expressed as a function of stretch, i.e. $k = \lambda - \lambda^{-1}$ with $\lambda > 1$. The strain invariants are defined by $I_1 = I_2 = k^2 + 3$.

There are several strain-energy density functions that are used to describe the mechanical behavior of hyperelastic materials. Here, for comparison purpose, Mooney–Rivlin, Yeoh, Ogden, Lopez-Pamies and Gent models are used.

1. *Mooney–Rivlin model*
One of the first strain energy function was proposed by Mooney and extended by Rivlin. The well-known Mooney–Rivlin model is given by [8]:

$$\Psi_{MR} = C_{10}(I_1 - 3) + C_{01}(I_2 - 3), \tag{3}$$

with the initial shear modulus equal to $\mu = 2(C_{10} + C_{01})$.

$$\sigma_{11} = \mu(\lambda^2 - \lambda^{-2}), \tag{4}$$

$$\sigma_{12} = \mu k. \tag{5}$$

2. *Yeoh model*
The model proposed by Yeoh, which is referred to as generalized neo-Hookean materials, is based on three terms that depends only on the first strain invariant. The Yeoh model is defined by [9]:

$$\Psi_Y = C_1(I_1 - 3) + C_2(I_1 - 3)^2 + C_3(I_1 - 3)^3, \tag{6}$$

where C_1, C_2 and C_3 denote material parameters. The initial shear modulus is equal to $\mu = 2C_1$.

$$\sigma_{11} = 2(\lambda^2 - \lambda^{-2})[C_1 + 2C_2(\lambda^2 + \lambda^{-2} - 2) + 3C_3(\lambda^2 + \lambda^{-2} - 2)^2], \tag{7}$$

$$\sigma_{12} = 2k[C_1 + 2C_2(\lambda^2 + \lambda^{-2} - 2) + 3C_3(\lambda^2 + \lambda^{-2} - 2)^2]. \tag{8}$$

3. *Ogden model with 1 term*
An alternative model for hyperelastic materials was postulated by Ogden, where the strain energy is a function of the principal stretches, instead of the first strain invariant. The Ogden model is expressed by [11]:

$$\Psi_O = \sum_{p=1}^{N} \frac{\mu_p}{\alpha_p}(\lambda_1^{\alpha_p} + \lambda_2^{\alpha_p} + \lambda_3^{\alpha_p} - 3), \tag{9}$$

where the initial shear modulus is $\mu = \frac{1}{2}\sum_{p=1}^{N}\mu_p\alpha_p$ with $\mu_p\alpha_p > 0$. Considering only one term of series, the Cauchy stress components are

$$\sigma_{11} = \mu_1(\lambda^{\alpha_1} - \lambda^{-\alpha_1}), \tag{10}$$

$$\sigma_{12} = \mu_1 \frac{\lambda}{1 + \lambda^2}(\lambda^{\alpha_1} - \lambda^{-\alpha_1}), \tag{11}$$

with $\lambda = \frac{k + \sqrt{k^2 + 4}}{2}$ (> 1) and $\mu = \frac{\mu_1\alpha_1}{2}$.

4. *Ogden model with 2 terms*
Taking into consideration the first two terms of the series [11]:

$$\sigma_{11} = \mu_1(\lambda^{\alpha_1} - \lambda^{-\alpha_1}) + \mu_2(\lambda^{\alpha_2} - \lambda^{-\alpha_2}), \tag{12}$$

$$\sigma_{12} = \frac{\lambda}{1 + \lambda^2}[\mu_1(\lambda^{\alpha_1} - \lambda^{-\alpha_1}) + \mu_2(\lambda^{\alpha_2} - \lambda^{-\alpha_2})], \tag{13}$$

with $\mu = \frac{\mu_1\alpha_1 + \mu_2\alpha_2}{2}$.