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A residual-based Gaussian process model framework for finite element model updating

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ABSTRACT

A residual-based Gaussian process model (GPM) framework is proposed for finite element model updating (FEMU). The core idea of the proposed method is that GPM is adopted to characterize the relationship between the residual and the selected parameters. Within the residual-based GPM framework, the powerful variance-based global sensitivity analysis can be analytically implemented for parameter selection, and the rate of convergence of the optimization process is accelerated substantially by providing the analytical gradient and Hessian information. A real-world arch bridge is presented to illustrate the proposed residual-based GPM framework and verify its feasibility in FEMU.

1. Introduction

Researchers in engineering area make extensive use of finite element model (FEM) for understanding real-world structures of interest and for decision-making (usage, maintenance, operations, etc.). Specifically, FEM enables engineering researchers to facilitate the model-based tasks, including but not limited to damage diagnosis, structural health monitoring, structural safety and risk assessment. Accordingly, a high-fidelity FEM that precisely represents the actual behavior of structures is of great importance. FEM is developed based on the engineering design blueprints, essentially an idealization of a real structure, which is likely to misrepresent the as-built real structure. As a result, the analytical results obtained from the idealized FEM often differ from those measured from a real structure. FEM updating (FEMU) aims to improve the agreement between the analytical and measured results by calibrating uncertain model parameters. The basic idea behind FEMU is that structural responses including dynamic and static ones are the functions of model parameters and thus adjusting the uncertain model parameters will result in the corresponding changes to structural responses, leading to an increase in correlation between analytical responses and their experimental counterparts.

In the past several decades, a great deal of research work has been dedicated to updating FEM in light of test results in order to precisely model the real-world structures. Comprehensive literature surveys on state-of-the-art in FEMU techniques can be found in Refs. [1–3]. Based on whether the methods for FEMU modify the elements of the system matrices (mass, stiffness and possibly damping matrices) directly or tune model parameters (e.g., structural geometric and material parameters) iteratively, they can be broadly classified as either direct or iterative. The direct method, which is a one-step approach for performing FEMU, first appears in the field of FEMU. Although the direct method is quite computationally-efficient and yields an exact agreement between analytical and measured responses as well, it is still limited in its applicability due to the loss of physical meaning in the resulting system matrices. The updated system matrices fail to preserve the features of sparseness, positive-definiteness, symmetry, etc. As a consequence, applications of the direct method to FEMU have seldom been explored over recent years. The iterative method (also called sensitivity-based or parametric method) involves using sensitivity technique to calibrate model parameters related to structure, which is typically a constrained optimization problem. In contrast to the direct method, the iterative method has the advantage of guaranteeing the physical meaning of the updated system matrices, and, moreover, the updated results can be well interpreted by the corrected model parameters. Therefore, the iterative method has become mainstream in the field of FEMU.

Although appropriate for FEMU, the iterative method remains computationally-intensive since it needs a large number of model evaluations to find the most optimal set of parameters ensuring the global minima of the objective function. The complex physical systems are usually modeled by the high-resolution FEMs involving

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up to tens or hundreds of millions of elements using commercial finite element analysis (FEA) packages (e.g., ANSYS), which may require many minutes, hours or even days for a single run. For example, it is reported that Ford Motor Company spends about 36–160 h in carrying out one crash simulation on a full-size passenger car [4]. When adopted to update the high-fidelity FEM of the complex physical systems, the iterative method may tend to be neither affordable nor feasible. Furthermore, the high-resolution FEM is constructed on the platform of the FEA packages, while the advanced optimization techniques (e.g., evolutionary algorithms and multi-objective optimization scheme), which can be easily implemented or available in numerical software (e.g., MATLAB), are may not offered by these FEA packages. To link FEA and numerical software together, one has to develop an interface between them, which will make the process of FEMU more complicated and also increase the computational burden because of repeatedly calling FEA package platform from numerical software platform.

Recently, metamodel is widely used as the surrogate model of the time-consuming FEM in order to circumvent the issue of high computational cost in FEMU. Among various metamodeling approaches, response surface method (RSM) is early and widely used for FEMU because of its appealing strengths, such as simple algebraic structure, easy implementation, and low effort. Schultze et al. [5] propose the use of RSM in feature extraction, parameter effect assessment, and nonlinear model updating. Steenackers and Guillaume [6] employ the RSM in FEMU considering measurement uncertainty. Ren and his colleagues [7,8] study the feasibility of RSM in the real-world bridge structure model updating using dynamic and static properties, respectively. Fang and Perera [9] present a RSM-based FEMU scheme for damage identification using D-optimal design. Shahidi and Pakzad [10] develop an improved RSM that is applicable to both linear and nonlinear FEMU. As a powerful alternative to metamodel, Gaussian process model (GPM), also termed *Kriging* process, has been exponentially applied in a variety of engineering problems, including design optimization [11,12], uncertainty quantification [13–16], stochastic finite element analysis [17,18], global sensitivity analysis [19–21], to name but a few. GPM possesses the following main admirable merits: (1) the data-driven feature of GPM enables it not to be restricted to a certain algebraic structure of the input–output relationship and guarantees the high flexibility in modeling a complex physical system; (2) GPM allows for assessing the uncertainty of the predictions by providing not just a predicted expectation but also the associated prediction variance; and (3) for the deterministic computer code simulation (e.g., FEM), GPM is able to yield exact predictions exactly over the observed data. Although GPM has become an attractive metamodeling tool for various engineering applications, little work has been done in applying GPM to FEMU. To our best knowledge, the limited research exploring the application of GPM to FEMU can be found in the works of Khodaparast et al. [22], Erdogan et al. [23], and Wan and Ren [24]. This study proposes a residual-based GPM framework for FEMU, which is different from the traditional metamodeling approach. To be specific, the traditional metamodeling approach refers to approximating the relationship between the selected parameters and structural responses, which can be either dynamic or static, whereas the proposed residual-based GPM approach aims at mapping the relationship between the selected parameters and the residual that will be minimized for the subsequent FEMU.

In this paper, we propose a residual-based GPM framework for FEMU. The core of this method is twofold: (1) GPM is used as the surrogate model; and (2) the residual between FEM-derived and measured responses instead of FEM-derived response is chosen as the output of the GPM. Within the residual-based GPM framework, our work consists of three main contributions in the following aspects:

- We propose the use of residual as the target response in metamodel construction, so only one metamodel will be required for FEMU. In contrast, the traditional metamodeling approach formulates the metamodel for each structural response, which means that the number of metamodel is the same as the number of structural responses. Each metamodel corresponds to the relationship between one certain type of the measured response and the selected parameters, so the residual is the composite function of the selected parameters. For example, assume we have 5 measured modal frequencies and 2 measured displacements, and then we have to establish a total of 7 metamodels associated with these measured responses. Therefore, our proposed residual-based metamodeling approach is superior to the traditional method in terms of the computational efficiency.
- Within residual-based GPM framework, we present an analytical implementation of variance-based global sensitivity analysis (GSA) for parameter selection. According to the first-order and total sensitivity indices (SIs) of the powerful variance-based GSA, we can have a deep understanding of how the selected parameters quantitatively influence the residual, thereby determining which parameters should be selected for being updated. This present analytical variance-based GSA based on GPM is more efficient and accurate than the Monte Carlo simulation (MCS) in conjunction with GPM because it implements variance-based GSA in an analytical manner.
- It is well known that FEMU procedure is in essence a constrained optimization problem, that is finding the global minimum of the objective function subject to constraints. Within the aid of GPM, the constrained optimization task may be easily carried out using the optimization toolbox available in MATLAB. We derive the closed form expressions for the first derivative (i.e., gradient) and second derivative (i.e., Hessian) of the objective function w.r.t. parameters. The analytical gradients can be utilized to accelerate the optimization process dramatically so that the model updating procedure becomes more efficient.

2. Residual-based GPM framework for FEMU

2.1. Gaussian process model

The probabilistic, non-parametric GPM is derived from a Bayesian setting, which allows for uncertainty quantification of the predictions. GPM treats model outputs as a random function with the associated probability distribution modeled through a Gaussian process prior; based on the maximum likelihood estimate of the training set, a Gaussian process prior combined with Gaussian likelihood results in a posterior Gaussian process over prediction at a new point. GPM is completely governed by its mean function and covariance function. The zero mean function is chosen because of the lack of prior knowledge for the overall trend of the latent function [25] and also for the sake of the simplified GPM formulation. On the other hand, we adopt the squared exponential covariance function below, which maintains an appealing property in that it leads to a smooth, infinitely differentiable function.

$$C(x_i, x_j') = \sigma^2 \exp\left(-\frac{1}{2}\sum_{k=1}^{d}\left(\frac{x_i^k - x_j^k}{\theta_k}\right)^2\right) \tag{1}$$

where $x_i^k(x_j^k)$ represents the k-th component of $x_i(x_j)$, in which $x_i = (x_i^1, x_i^2, \ldots, x_i^d)$ is a d-dimensional input vector for observation i. The covariance function parameters $\Theta = (\theta_1, \ldots, \theta_d, \sigma^2)$ are called hyperparameters.

Assume we have a training set with n observations, $\mathrm{D} = (\mathrm{X}, \mathrm{T})$, where $\mathrm{X} = (x_1^T, x_2^T, \ldots, x_n^T)^T$ and $\mathrm{T} = (t_1, t_2, \ldots, t_n)^T$. The predictive