Computers and Structures 122 (2013) 88–100

Contents lists available at SciVerse ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

Anisotropic adaptive meshing and monolithic Variational Multiscale method for fluid–structure interaction

E. Hachem [a,*], S. Feghali [a], R. Codina [b], T. Coupez [a]

[a] Center for Material Forming (CEMEF), MINES-ParisTech, UMR CNRS 7635, Sophia-Antipolis, France
[b] Universitat Politecnica de Catalunya (UPC), Jordi Girona 1-3, 08034 Barcelona, Spain

ARTICLE INFO

Article history:
Received 7 September 2012
Accepted 3 December 2012
Available online 5 January 2013

Keywords:
Stabilized finite elements
Monolithic approach
Rigid body motions
Incompressible flows
Fluid–structure interactions
Anisotropic meshing

ABSTRACT

This paper presents a monolithic formulation framework combined with an anisotropic mesh adaptation for fluid–structure interaction (FSI) applications with complex geometry. The fluid–solid interfaces are captured using a level-set method. A new *a posteriori* error estimate, based on the length distribution tensor approach and the associated edge based error analysis, is then used to ensure an accurate capturing of the discontinuities at the fluid–solid interface. It enables to calculate a stretching factor providing a new edge length distribution, its associated tensor and the corresponding metric. The optimal stretching factor field is obtained by solving an optimization problem under the constraint of a fixed number of edges in the mesh. The presence of the structure will be taken into account by means of an extra stress tensor in the Navier–Stokes equations. The system is solved using a stabilized three-field, stress, velocity and pressure finite element (FE) formulation. It consists in the decomposition for both the velocity and the pressure fields into coarse/resolved scales and fine/unresolved scales and also in the efficient enrichment of the extra constraint. We assess the accuracy of the proposed formulation by simulating 2D and 3D time-dependent numerical examples such as: falling disk in a channel, turbulent flows behind an airfoil profile and flow behind an immersed vehicle.

1. Introduction

Fluid–structure Interaction (FSI) describes a wide variety of industrial problems arising in engineering, technology and biomechanics. Due to the high complexity of these problems, FSI simulations are nowadays the focus of numerous investigations and various approaches are proposed to treat them.

Two main approaches for the simulation of FSI problems are still gaining attention lately: partitioned and monolithic approaches. The partitioned approaches allow the use of a specific solver for each domain. The fluid and the structure equations are alternatively integrated in time and the interface conditions are enforced asynchronously. The difficulty remains in transferring the informations between the codes. The coupling between the two phases can be enforced using different schemes: weakly or strongly coupled versions. The former approach manages with just one solution of either field per time step but consequently lack accurate fulfillment of the coupling conditions. The latter requires sub-iterations [1–6]. It is accurate and quite efficient but presents an inherent instability depending on the ratio of the densities and the geometry of the domain [7]. For 3D problems, the numerical cost can increase drastically. Alternatively, authors in [8] propose an immersed particle method able to handle complicated FSI problems including cracking and perforation.

Monolithic methods are still of interest due to their capability to treat the interaction of the fluid and the structure at the interface synchronously [9–11]. The continuity at the interface is obtained naturally and there is no need to enforce it. They impose the use of an appropriate unique constitutive equation describing both the fluid and the solid domains. Interface tracking between the two different domains can be completed by Immersed Boundary (IB) methods [12] where the interface is convected in a Lagrangian way. Other methods such as the fictitious domain method [12,13] treat the coupling between the domains by applying a constraint across the body using a Lagrange multiplier. These constraints may lead to uncoupled physics in the different subdomains of the problem (in the fluid and the solid, for example), yielding inconsistencies when the subdomains evolve in time. This problem may be solved using the so called Fixed-mesh ALE formulation introduced in [14] (see also [15,16] for applications to fluid–structure interaction problems and rigid bodies floating in fluids). Likewise and for more complex problems using the ALE formulation, the authors in [17] proposed a mesh adaptivity procedure for fluid–structure interactions capable of handling high gradients in the solution, boundary layer effects and large structural deformations.

* Corresponding author. Tel.: +33 (0)4 93 95 74 58; fax: +33 4 92 38 97 52.
E-mail addresses: Elie.hachem@mines-paris tech.fr (E. Hachem), Stephanie.El_Feghali@mines-paristech.fr (S. Feghali), ramon. codina@upc.edu (R. Codina), Thierry.Coupez@mines-paristech.fr (T. Coupez).

0045-7949/$ - see front matter © 2012 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.compstruc.2012.12.004

In this paper, we focus on a monolithic formulation where the complete problem is written in a fully Eulerian framework and the fluid and solid phases are separated by a level-set function. This was started in [18,19] for simulating the interactions between a fluid and fixed solids. The solid was simply treated as a region with high viscosity and the mesh near the fixed interfaces was refined a priori and before the simulation. The criterion was obviously the levelset function.

In this work, we develop a new monolithic approach that differs from the previous developments in two main aspects. The first one is related to the way we adapt the mesh in particular for moving rigid bodies and the second focuses more on how we treat the solid regions in the Navier–Stokes equations. Therefore, we introduce a new dynamic anisotropic mesh adaptation method to deal properly with moving interfaces which still is a key challenge in most of the monolithic approaches. Inspired from the work in [20], we propose an extension of the edge based error estimation to combine the simultaneous adaptivity to the interface and to the velocity field using one simple global vector field. With such an advantage, it becomes a very useful and practical tool for a wide range of FSI problems. The second point concerns more the development of a three-field stabilized finite element method and its implementation aspects for modeling the interaction between the fluid (laminar or turbulent) and the rigid bodies (fixed or moving). The presence of the structure will be then taken into account by means of an extra stress tensor in the Navier–Stokes equations.

In Sections 2–4, we first consider the level-set function, commonly employed in the simulation of multiphase flows [21], used to distinguish the phases. It allows to easily deal with very complex geometries, large structural deformations and free movements of the structure within a flow domain. However, the level-set intersects the elements arbitrarily and lacks the ability to reproduce the interfaces of complex geometries (i.e. sharp corners). Therefore, we combine it with anisotropic mesh adaptation. An *a posteriori* edge based spatial error indicator relying on the length distribution tensor approach is presented in section 3. The anisotropic adaptation involves building a mesh based on a metric map. It provides both the size and the stretching of elements in a very condensed information data. Working on a nodal based metric, an anisotropic mesh adaptation procedure is obtained under the constraint of a fixed number of nodes. With such an advantage, it becomes a very useful and practical numerical tool. Such an algorithm allows the creation of extremely stretched elements along the interface, which is an important requirement for FSI problems with high density ratios.

In Section 4, we then present the development of the FE solver. The rigid immersed body is treated using the Navier–Stokes solver under constraints of imposing the nullity of the deformations by means of a Lagrange multiplier. The system is solved using a new Variational Multiscale FE method. Thus we propose to extend the decomposition for both the velocity and the pressure fields into coarse/resolved scales and fine/unresolved scales, needed to deal with convection dominated problems and pressure instabilities, with an efficient enrichment of the extra constraint. This choice of decomposition is shown to be favorable for simulating flows at high Reynolds number and to remove spurious oscillations at the interface due to the high discontinuity in the material properties. We retain in this work the advantages of using the P1 finite elements approximation regarding the accuracy and the computational cost, especially for 3D real applications.

The capability of the developed finite element method in handling extremely stretched elements and in producing very satisfactory results is highlighted in Section 5 through different numerical tests. We show that the proposed anisotropic meshing technique is well suited for these fluid–structure interaction problems and could be embedded into different FSI techniques such as [22–24]. This is due to the fact that the latter method takes into account multicomponent fields simultaneously (tensors, vectors, scalars) characterizing the structure and the physics of the problem. For instance it includes the velocity norm, the velocity components and the Level-Set function combined into one single metric field. In Section 6, we give our concluding remarks.

2. Construction of an anisotropic mesh

In this section, we retrace the main steps of the adaptive procedure used to immerse and to represent different complex geometries inside a unique mesh. First we compute the signed distance function (level-set) of a given geometry to each node of the mesh, then we refine anisotropically the mesh at the interface and finally we mix and attribute the physical properties of each domain using appropriate laws. This procedure is repeated iteratively for moving solids.

2.1. Level-set function

A signed distance function of an interface Γ_{im} is used to localize the interface of the immersed body and initialize the desirable properties on both sides of the latter. At any point $\mathbf{x}$ of the computational domain Ω, the level-set function α_{im} corresponds to the signed distance from Γ_{im}. In turn, the interface Γ_{im} is given by the iso-zero of the function α_{im}:

$$\begin{cases} \alpha_{\text{im}}(\mathbf{x}) = \pm d(\mathbf{x}, \Gamma_{\text{im}}), & \mathbf{x} \in \Omega, \\ \Gamma_{\text{im}} = \{\mathbf{x}, \alpha_{\text{im}}(\mathbf{x}) = 0\}. \end{cases} \tag{1}$$

In this paper, a sign convention is used: $\alpha_{\text{im}} \geqslant 0$ inside the solid domain defined by the interface Γ_{im} and $\alpha_{\text{im}} < 0$ outside this domain. Further details about the algorithm used to compute the distance are available in [25]. It is also possible to use functions smoother than $d(\mathbf{x}, \Gamma_{\text{im}})$ far from Γ_{im} (see for example [26]).

2.2. Edge based error estimation

An *a posteriori* error estimate based on the length distribution tensor approach and the associated edge based error analysis [20] is presented. It enables to calculate a stretching factor providing a new edge length distribution, its associated tensor and the corresponding metric. The optimal stretching factor field is obtained by solving an optimization problem under the constraint of a fixed number of edges in the mesh. In this work, we emphasis the application of this new technique to multi-domain problems. Therefore, for addressing a high contrast in the physical parameters, we propose an extension of the *a posteriori* estimation. It combines the simultaneous adaptivity to the level-set scalar field and to the velocity field without increasing the complexity of the computation or intersecting different metrics. Using this approach, the adaptivity will also focus on the change of direction rather than the intensity of the velocity. This is clearly shown behind the obstacle in Fig. 1, whereas the adaptation on the level-set function renders extremely stretched elements along the fluid–solid interface. With such a method, we can provide a very useful and practical tool for the simulation of complex FSI problems. In the following subsections, details of the adaptivity approach will be discussed.

We consider a variable $u \in \mathcal{C}^2(\Omega) = \mathcal{V}$ and $\mathcal{V}_h$ a simple P^1 finite element approximation space: $\mathcal{V}_h = \{w_h \in \mathcal{C}^0(\Omega),\ w_h|_K \in P^1(K),\ K \in \mathcal{K}\}$ where $\Omega = \bigcup_{K\in\mathcal{K}} K$ and K is a simplex (segment, triangle, tetrahedron, ...).

We define $\mathbf{X} = \{\mathbf{X}^i \in \mathbb{R}^d,\ i = 1, \ldots, N\}$ as the set of nodes of the mesh and we denote by U^i the nodal value of u at $\mathbf{X}^i$ and we let Π_h be the Lagrange interpolation operator from $\mathcal{V}$ to $\mathcal{V}_h$ such that: $\Pi_h u(\mathbf{X}^i) = u(\mathbf{X}^i) = U^i,\ \forall i = 1, \ldots, N$. As shown in Fig. 2, we denote