Computers and Structures 122 (2013) 192–201

Contents lists available at SciVerse ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

# Chaotic oscillations of long pipes conveying fluid in the presence of a large end-mass

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ARTICLE INFO

*Article history:*
Received 30 May 2012
Accepted 14 February 2013
Available online 29 March 2013

*Keywords:*
Pipes conveying fluid
End-mass
Nonlinear dynamics
Chaos

ABSTRACT

Copeland and Moon's experimental results for a long pipe conveying fluid in the presence of a relatively large end-mass have displayed some truly fascinating dynamical behavior. Numerical studies, on the other hand, have all dealt with shorter pipes and smaller end-masses, mainly because the numerical convergence of the theoretical results for long pipes with large end-masses is problematic. In this paper, numerical results are presented for Copeland and Moon's system parameters, reproducing some of the rich dynamics they obtained, including coupled planar and pendular oscillations, planar oscillations rotating through a finite angle, and planar motions rotating clockwise or counter-clockwise.


## 1. Introduction

The first studies on cantilevered pipes conveying fluid with attached masses were undertaken using linear models [1]. This linear work was later continued by Jendrzejczyk and Chen [2] for a pipe with a mass attached at the free end (here referred to as an "end-mass") and by Sugiyama et al. [3]. These studies showed that the additional mass(es) could either stabilize or destabilize the system vis-à-vis the plain pipe, depending on the system parameters and location of the additional mass(es) [see [4], Section 3.6.3].

Nonlinear studies of a pipe with an end-mass started mainly after nonlinear equations for a plain pipe (i.e., without additional masses or springs attached to the pipe) were derived by several researchers for planar (2-D) motion (e.g., [5,6]) and for three-dimensional (3-D) motion (e.g., [7–9]). A plain pipe loses stability by a Hopf bifurcation at a critical flow velocity and undergoes limit cycle oscillations for larger flow velocities. No further bifurcations are observed for increasing flow.

The first, very interesting nonlinear study on pipes with an end-mass was the experimental work by Copeland and Moon [10]. The experiments were conducted with particularly long, vertically hanging, cantilevered elastomer pipes, fitted at the free end with end-masses of different sizes and showed extremely rich dynamical behavior, as summarized in Fig. 1, where $\Gamma = m_e/[(M+m)L]$ is a dimensionless end-mass parameter, with $m_e$ being the end-mass, $M$ the mass of the fluid per unit length, $m$ that of the pipe per unit length, and $L$ the pipe length; $u_g = U/(gL)^{1/2}$ is the dimensionless flow velocity used by Copeland and Moon. In addition to planar and orbital (rotary) motions, an extraordinary array of geometrically more complex motions was discovered. In all cases with an end-mass, for sufficiently high flow velocity the motions became chaotic. In at least some cases, the quasiperiodic route to chaos was found to be followed. In the analytical part of the study, which was not wholly successful [13], the long vertical pipes were modeled as hanging strings.

Païdoussis and Semler [12] studied both theoretically and experimentally the dynamics of more modestly long hanging elastomer cantilevers. In these experiments, the end-mass parameters were considerably smaller than in Copeland and Moon's experiments. Interesting observations were made in this case also. Planar flutter was followed by a secondary bifurcation as the flow velocity was increased, which could be identified with a sudden and substantial increase in the frequency, accompanied by a peculiar mode of oscillation with a seemingly stationary node around the pipe mid-length. For higher flow velocities, the motion eventually became chaotic and three-dimensional. Païdoussis and Semler's [12] experimental results were compared to the theoretical ones, obtained with their 2-D model. The Hopf and secondary bifurcations were reasonably well predicted, as was the transition to chaos. See [4, Section 5.8.3] for an extensive review of nonlinear work on pipes with an end-mass.

A 3-D version of the nonlinear equations of motion of Semler et al. [6] has been derived by Wadham-Gagnon et al. [14]. These equations have been used successfully [15] to study the three-dimensional behavior of a pipe with an end-mass and with physical properties of the pipe as in Païdoussis and Semler's

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0045-7949/$ - see front matter 
http://dx.doi.org/10.1016/j.compstruc.2013.02.005

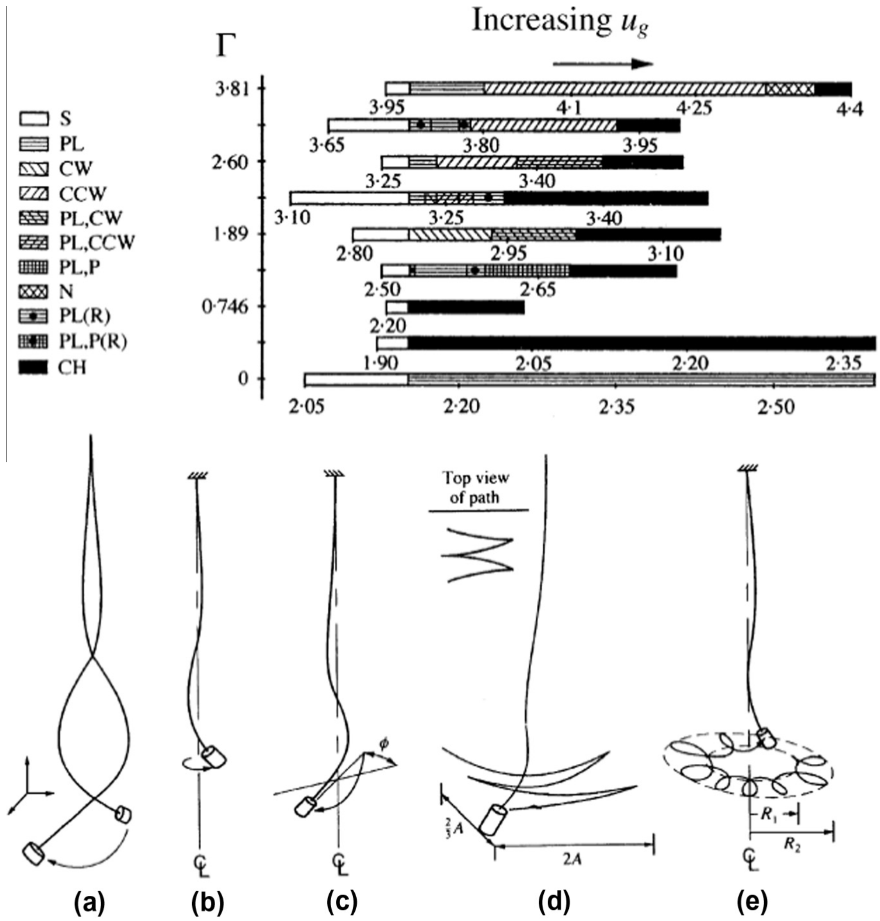


**Fig. 1.** Transition from equilibrium to chaos for 3-D motions of the system for various end-masses. Top: the ranges of various oscillatory states in terms of increasing $u_g$ for different end-masses, $\Gamma$. S: stationary pipe; PL: planar oscillation; CW: clockwise rotating motion; CCW: counter-clockwise rotating motion; PL, CW: clockwise rotating planar oscillation; PL, CCW: counter-clockwise rotating planar oscillation; PL(R): planar oscillation rotating through a finite angle; PL, P: coupled planar and pendular oscillation; PL, P(R): coupled oscillation and pendular oscillation rotating through a finite angle; N: nutation; CH: chaos. Bottom: sketches of various periodic motions. (a) PL; (b) CCW; (c) PL(R); (d) PL, P; (e) N [10].

experiments [12]. In the same paper, it is shown that convergence of the 3-D theoretical results for cases with large gravitational parameter ($\gamma$; proportional to the length cubed) and end-mass parameter ($\Gamma$) is not easily obtainable. In the present paper, the 3-D study of pipes with end-masses is extended to the range of the pipes used in Copeland and Moon's experiments [10]. While there might not be any direct application of exactly the problem studied here, there are various engineering applications in which dynamic flow-induced instabilities are observed. There is currently a lot of interest in the dynamics of very long (kilometer-long) pipes used in solution-mined salt caverns, used for underground storage of hydrocarbons [11]. The fundamental knowledge gained from studies such as the one discussed in this paper can help in understanding similar instabilities in other, sometimes more complicated, systems.

## 2. Theoretical model

The equations of motion have been derived by Wadham-Gagnon et al. [14] for a general case where there are some intermediate springs as well as an end-mass attached to the pipe. For the present study (Fig. 2), all terms related to these springs have been deleted. In the derivation, the Lagrangian coordinates ($X_0$,$Y_0$,$Z_0$), which label specific particles at the original equilibrium state of the pipe, are related to the Eulerian coordinates ($x$,$y$,$z$) through the displacement ($u, v, w$) of any material point as

$$x = X_0 + u, \quad y = Y_0 + v, \quad z = Z_0 + w. \tag{1}$$

A curvilinear coordinate $s$, along the length of the pipe is introduced. The pipe is assumed to be inextensible, which leads to the following relation:

$$\left(1+\frac{\partial u}{\partial s}\right)^2 + \left(\frac{\partial v}{\partial s}\right)^2 + \left(\frac{\partial w}{\partial s}\right)^2 = 1. \tag{2}$$

To derive the equation of motion, a modified version of Hamilton's principle is used, where the right-hand side accounts for the energy gain or loss at the free end of the pipe [16,17]:

$$\delta\int_{t_1}^{t_2} L\mathrm{d}t + \int_{t_1}^{t_2} \delta W \mathrm{d}t = \int_{t_1}^{t_2} \left\{ MU\left[\frac{\partial \mathbf{r}_L}{\partial t} + U\boldsymbol{\tau}_L\right]\cdot\delta\mathbf{r}_L \right\}\mathrm{d}t, \tag{3}$$

in which, $L = T_p + T_p - V_p - V_f$ is the Lagrangian of the system; $T$ and $V$ refer to the kinetic and potential energy, respectively; subscripts $p$ and $f$ refer to the pipe and the fluid, respectively; $\delta W$ is the virtual work due to the forces not included in the Lagrangian, and the right-hand side is the virtual momentum transport at the end of the pipe; $r$ is a position vector, and $\tau$ is the tangent vector at any point of the pipe:

$$\mathbf{r} = (s+u)i + vj + wk, \tag{4}$$

$$\boldsymbol{\tau} = \left(1+\frac{\partial u}{\partial s}\right)i + \frac{\partial v}{\partial s}j + \frac{\partial w}{\partial s}k. \tag{5}$$

In the derivation, it is assumed that the displacements in the $y$ and $z$ directions are of order $\varepsilon$ and the nonlinear terms up to the third