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Bifurcations and chaotic motions of a curved pipe conveying fluid with nonlinear constraints

Ni Qiao, Wang Lin *, Qian Qin

Department of Mechanics, Huazhong University of Science and Technology, Wuhan 430074, PR China

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Abstract

This paper investigates the bifurcations and chaotic motions of a fluid-conveying curved pipe restrained with nonlinear constraints. The nonlinear equation of motion for the curved pipe is derived by forces equilibrium on microelement of the system under consideration. Depending on the nonlinear equation of motion and the corresponding boundary conditions for the curved pipe, the DQM (differential quadrature method) is introduced to formulate the discrete forms of the equation of motion for the system, which is then solved by numerical methods. Calculations of phase-plane portraits, time history diagrams, PS (Power Spectrum) diagrams, bifurcation diagrams and Poincaré maps of the oscillations establish clearly the existence of the chaotic motions. In addition, the result shows the route to chaos for the pipe is via period-doubling bifurcations, which is affected definitively by several parameters of the system.
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Keywords: Curved pipe conveying fluid; Bifurcation; Chaotic motion; Nonlinear constraint; DQM

1. Introduction

It is well known that the study of pipes conveying fluid was one of the attractive subjects in dynamics during the previous decades. Thus, many investigators have studied on the linear vibrations of pipes conveying fluid, both for the straight and curved pipes.

With the recent developments of the theory of possible on nonlinear dynamical systems and chaos, much attention has been paid to the study of possible existence of chaotic motions in some modified systems with strong nonlinear factors. Thus the chaos of straight pipes has been studied quite extensively [1–4].

As for curved pipes, they were started studying in recent decades because of their greater complexity than straight ones. The work on curved pipes is mainly placed on the linear problem of the systems. Most research work was concentrated on the derivation of the equations of motion and the linear vibration behavior for curved pipes. Chen [5–7] has studied the governing equations of uniformly curved pipes, both for the in-plane and out-of-plane vibrations; Misra et al. [8,9] calculated a curved pipe with complex shape by finite element method in 1988; Dupuis and Rousselet [10] utilized a transfer matrix method to investigate the dynamics of curved pipes; next, Aithal and Gipson [11] studied a semi-circular pipe conveying fluid by an analytical method; recently, Ni and Huang [12] used the DQM to study the stability of a curved pipe. Nevertheless, the literature on nonlinear vibration of fluid-conveying curved pipes is quite limited. Ko and Bert [13] considered the first-order nonlinear interactions between the pipe structure and the flowing fluid and formulated the governing equations of motion for the in-plane vibrations of a circular-arc pipe containing flowing fluid. In 1992, Dupuis and Rousselet [14] derived the nonlinear differential equations of motion of a fluid-conveying pipe by making use of the Newtonian approach. From the literature survey, it was hardly found such a work that deals with the chaotic motions of curved pipe conveying fluid. The origination

* Corresponding author. Tel.: +86 27 87543438; fax: +86 27 87543238.
E-mail address: wanglinfliping@sohu.com (W. Lin).

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and control mechanism of the chaotic motions of such a structure are not revealed yet. Similarly to a cantilever straight pipe, a cantilever curved pipe under certain conditions may show possible existence of chaotic motions. Therefore, the present work is trying to study the chaos and the route to chaos and to reveal the abundant dynamic behaviors of such a pipe system. In this paper, a cantilever curved pipe conveying fluid as a model, the motion of which is limited to in-plane, is restrained with nonlinear constraints close to its free-end. The analysis process is beginning with the foundation of the system's equation of motion, and then the equation of motion is carried out by numerical approaches: the discrete equation of motion is obtained by DQM and its solution is then obtained via Newton–Raphson method. The numerical analysis shows the chaotic motions of the curved pipe are induced in the parameter space of the system under certain conditions. Moreover, the route to chaos is shown to be through period-doubling bifurcations in some parameter regions. The results can provide further understanding of the dynamic behavior of the curved pipes conveying fluid.

2. Problem formulation

As shown in Fig. 1, the curved pipe is restrained with nonlinear constraints (or motion-limiting constraints) at the radius angle θ_c. The model under consideration consists of a uniformly curved pipe having a radius of curvature R, mass per unit length m, effective flexural rigidity EI, and internal perimeter S, conveying fluid of mass per unit length M, flowing with a constant velocity V, internal cross-sectional area A, the fluid pressure p, and subtended angle θ_0. This analysis is carried out for the in-plane vibration of such a fluid-structure interactive system. Several assumptions for the system are listed as follows:

- The fluid flow is incompressible and steady with a mean velocity V.
- The effect of external damping is small and is neglected here. The centerline of curved pipe is inextensible, thus

$$u = \frac{\partial w}{\partial \theta} \tag{1}$$

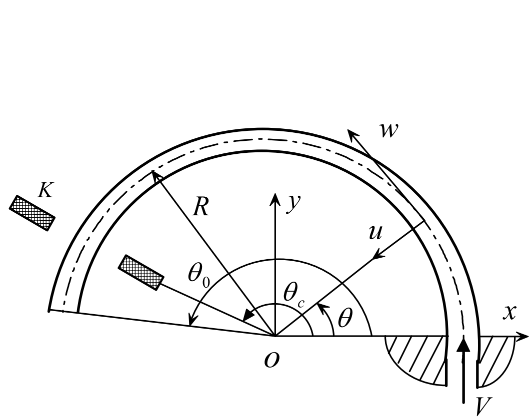

Fig. 1. A curved pipe conveying fluid.

- The effect of nonlinear constraints can be written as the restraining force

$$f = Ku^3\delta(\theta - \theta_c) \tag{2}$$

where $\delta(\cdot)$ is Dirac delta function; θ is the angular coordinate; K is the stiffness of the cubic spring which represents the effect of the nonlinear constraints; u and w denote the radial dispersion and tangential displacement, respectively. According to Ref. [5], for one element of the curved pipe (Fig. 2), the radius of curvature after deformation, is given by the flowing relation

$$\frac{1}{R'} = \frac{1}{R}\left(1 + \frac{u}{R} + \frac{1}{R}\frac{\partial^2 u}{\partial \theta^2}\right) \tag{3}$$

Let F, T and $\overline{M}$ be the internal shear force, normal force, and bending moment, respectively. The differential equations for rotational and translatory motions in radial and tangential directions of the element are

$$\frac{\partial \overline{M}}{\partial \theta} + RF = 0 \tag{4}$$

$$\frac{\partial F}{\partial \theta} + T - RQ - fR = mR\frac{\partial^2 u}{\partial t^2} \tag{5}$$

$$\frac{\partial T}{\partial \theta} + RSq - F = mR\frac{\partial^2 w}{\partial t^2} \tag{6}$$

where q is the shear stress on the internal surface of the pipe, and Q is the transverse force per unit length between pipe wall and fluid.

Similarly, consider the fluid element (Fig. 3). The accelerations of a fluid particle in tangential and radial directions are, respectively,

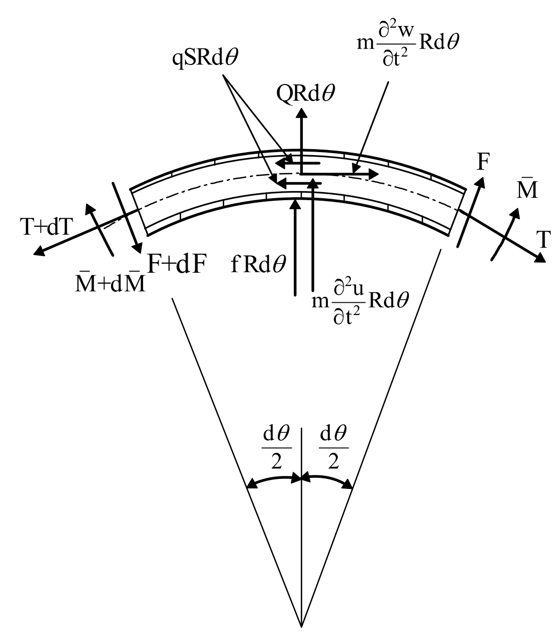

Fig. 2. Forces and moments acting on elements of the pipe.