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Free vibration analysis of rotating Euler beams at high angular velocity

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ABSTRACT

The natural frequency of the flapwise bending vibration, and coupled lagwise bending and axial vibration is investigated for the rotating beam. A method based on the power series solution is proposed to solve the natural frequency of very slender rotating beam at high angular velocity. The rotating beam is subdivided into several equal segments. The governing equations of each segment are solved by a power series. Numerical examples are studied to demonstrate the accuracy and efficiency of the proposed method. The effect of Coriolis force, angular velocity, and slenderness ratio on the natural frequency of rotating beams is investigated.

1. Introduction

Rotating beams are often used as a simple model for propellers, turbine blades, and satellite booms. The free vibration frequencies of rotating beams have been extensively studied [1–19]. Rotating beam differs from a non-rotating beam in having additional centrifugal force and Coriolis effects on its dynamics. The lagwise bending and axial vibration are coupled due to the Coriolis effects [8,15,18]. However, most studies neglected the Coriolis effects in the literature. It is well known that the beam sustains a steady state axial deformations (time-independent displacement) induced by constant rotation [20]. For the rotating uniform beam as shown in Fig. 1, the maximum steady axial strain occurs at the root of the beam and may be expressed as [15] $\varepsilon_{\max} = \bar{k}^2(R/L + 0.5)$, where $\bar{k} = \Omega L\sqrt{\rho/E}$ is a dimensionless angular velocity, R is the radius of the hub, L, ρ, and E are the length, density, and Young's modulus of the beam, respectively, Ω is the angular velocity of the hub. In practice, rotating structures are designed to operate in the elastic range of the materials. Thus, the allowable value of the maximum steady axial strain for the rotating beam should be smaller than the yield strain, which is much smaller than unity for most engineering material. In this sense, if the maximum steady axial strain is close to the yield strain, the corresponding angular velocity may be called high angular velocity. However, as mentioned in [15], the magnitudes of the steady state axial strain induced by the centrifugal force and the corresponding angular velocity are not checked in most literature. The dimensionless angular velocity used in most literature is $\bar{\eta}\bar{k}$, where $\bar{\eta}$ is the slenderness ratio of the beam. The difference of the maximum steady axial strains corresponding to the same value of $\bar{\eta}\bar{k}$ may be remarked for rotating beams with different slenderness ratio. Thus, the maximum steady axial strains corresponding to some angular velocity considered in many literatures are even larger than unity for rotating beam with small slenderness ratio, but are much smaller than the yield strain of most engineering material for very slender beam. In this study, if the maximum steady axial strain is much smaller than the yield strain, the corresponding angular velocity is regarded as low angular velocity. To the authors' knowledge, the study of the natural frequency for very slender rotating beam at high angular velocity is rather rare in the literature. The objective of this paper is to investigate the natural frequencies of the flapwise bending vibration, and coupled lagwise bending and axial vibration for very slender rotating Euler beam at high angular velocity using power series solution. However, the rotating beams with different slenderness ratio at different angular velocities are also investigated.

A number of methods based on the power series solution have been developed for determination of natural frequencies and mode shapes of rotating beams [2,6,9–19]. However, only the uncoupled bending vibration was considered in most methods based on the power series solution. It was asserted that only one single segment is needed for power series solution to obtain any modal frequency or mode shape for uniform beams or uniformly tapered beams in [17]. A similar statement was given in [18]. However, no results for slender rotation beams at high angular velocity were given in [17,18]. The assertion given in [17,18] may be correct if a computer can retain infinite number of significant digits to represent the result of an operation. However, any computer can only retain a finite number of significant digits to represent the result of an operation. The accuracy of the calculated natural frequency

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Nomenclature

A	cross-section area of rotating beam
E	Young's modulus
F_1	force in X_1 direction
F_3	force in X_3 direction
I	principal second moment of cross-section area
k	$\bar{k}/N$
$\bar{k}$	dimensionless angular velocity of the hub
K	$\overline{K}/N$
$\overline{K}$	dimensionless natural frequency of rotating beam
l	length of each segment
L	length of rotating beam
M	moment about negative X_2 axis
N	number of segments
$\bar{r}$	dimensionless radius of rotating hub
$\mathbf{r}$	position vector
$\ddot{\mathbf{r}}$	second time derivative of $\mathbf{r}$
R	radius of the hub
t	time
u	time dependent infinitesimal displacements in X_1 direction
U	dimensionless u
u_s	steady state axial deformations
w	time dependent infinitesimal displacements in X_3 direction
W	dimensionless w
β	setting angle of rotating beam
ε	axial strain
$\varepsilon_{\max}$	maximum steady axial strain of rotating beam
η	$\bar{\eta}/N$
$\bar{\eta}$	slenderness ratio of rotating beam
θ	rotation of beam cross-section
ω	natural frequency of rotating beam
Ω	angular velocity of hub

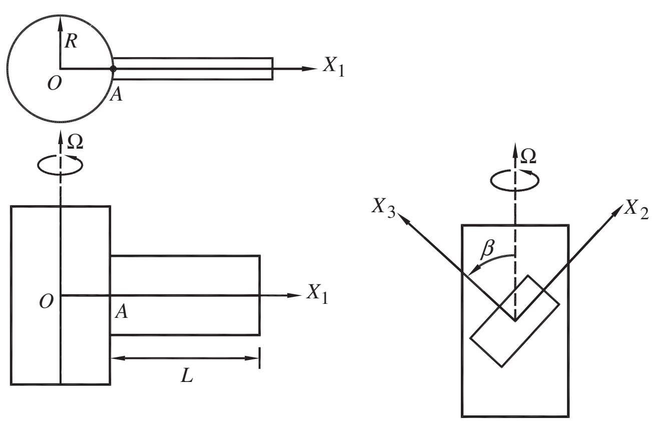

Fig. 1. A rotating Euler beam.

depends on the precision with which the computing facility operates, and a great number of terms in the power series solution does not necessarily result in a more accurate solution [12]. The authors use the power series method proposed in [15] and double precision computation to calculate the natural frequencies for slender rotating beams at high angular velocity. It is found that the rate of convergence of the power series solution is slower at higher angular velocity and the computation fail to converge when the angular velocity is higher than some value. In [12], it is found that to calculate the frequencies for rotating beam at high centrifugal tension using one segment and quadruple precision computation failed for some case due to arithmetic overflow. The failure may be attributable to the accuracy lost caused by the insufficient precision used in computation. It seems that the rate of convergence of the power series solution decreases and the degree of accuracy lost of the power series solution in computation increases with the increase of the dimensionless angular velocity $\bar{k}$. The power series solution using one segment with quadruple precision computation may be still not enough to get the frequencies for slender rotating beams at very high angular velocity. However, using quadruple or higher precision computation may be impractical. To alleviate the aforementioned numerical difficulties, in this study, a practical method based on the power series solution is proposed to solve the natural frequency of slender rotating beam at very high angular velocity.

In this study, the equations of motion for rotating Euler beam are derived by the d'Alembert principle and the virtual work principle. In order to capture all inertia effect and coupling between extensional and flexural deformation, the consistent linearization [21–23] of the fully geometrically non-linear beam theory [22,23] is used in the derivation. The governing equations for linear vibration of rotating beam are two coupled linear ordinary differential equations with variable coefficients. The rotating beam is subdivided into several equal segments. The solution of each segment is expressed as a power series with six independent coefficients. Substituting the power series solution of each segment into the corresponding boundary conditions at two end nodes of the rotating beam and the continuity conditions at common node between two adjacent segments, a set of homogeneous equations can be obtained. The natural frequencies may be determined by solving the homogeneous equations using the bisection method.

The dimensionless angular velocity corresponding to each segment is $\bar{k}/N$, where N is the number of segment. Subdividing the rotating beam into more segments can make the value of dimensionless angular velocity in the power series solution smaller. We believe that when the value of dimensionless angular velocity in the power series solution decrease, the rate of convergence of power series solution will increase, the accuracy lost in computation will decrease, and double precision computation will be sufficient to obtain natural frequency with high accuracy for slender rotating beams at very high angular velocity. This belief will be examined through numerical examples in the paper. Numerical examples are studied to investigate the effect of Coriolis force, rotary inertia, angular velocity, hub radius and slenderness ratio on the natural frequency of rotating beams. The frequency veering phenomenon [24] induced by the Coriolis force and the centrifugal force are also investigated.

2. Formulation

2.1. Description of problem

Consider a uniform Euler beam of length L rigidly mounted on the periphery of rigid hub with radius R rotating about its axis fixed in space at a constant angular velocity Ω as shown in Fig. 1. The deformation displacements of the beam are defined in a rotating rectangular Cartesian coordinate system which is rigidly tied to the hub. The origin of this coordinate system is chosen to be the intersection of the centroid axes of the hub and the undeformed beam. The X_1 axis is chosen to coincide with the centroid axis of the undeformed beam, and the X_2 and X_3 axes are chosen to be