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Finite element-based analysis of shunted piezoelectric structures for vibration damping

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Abstract

Piezoelectric patches shunted with passive electrical networks can be attached to a host structure for reduction of structural vibrations. This approach is frequently called "shunted piezo damping" and has the advantage of guaranteed stability and low complexity in implementation. For numerical treatment of such structures, a finite element modelling methodology is presented that incorporates both the piezoelectric coupling effects of the patches and the electrical dynamics of the connected passive electrical circuits. It allows direct computation of the achieved modal damping ratios as a major design criterion of interest. The damping ratios are determined from the eigenvalue problem corresponding to the coupled model containing piezoelectric structure and passive electrical circuit. The model includes local stiffening and mass effects as a result of the attached patches and, therefore, enables accurate prediction of the natural frequencies and corresponding modal damping ratios. This becomes crucial for choosing the patch thickness to achieve optimal modal damping for a given host structure. Additionally, structures with complex geometry or spatially varying material properties can easily be handled. Furthermore, the use of a finite element formulation for the coupled model of piezoelectric patches and a host structure facilitates design modifications and systematic investigations of parameter dependencies. In this paper, the impact of parameters of the passive electrical network on modal damping ratios as well as the variation of the patch thickness are studied. An application of this modelling method is realized by commercial software packages by importing fully coupled ANSYS© – models in MATLAB©. Afterwards, modal truncation is applied, the dynamic equations of the passive electrical network are integrated into the piezoelectric model and eigenvalue problems are solved to extract the increase in modal damping ratios. The numerical results are verified by experiments.

Keywords: Piezoelectric structure; Passive electrical network; Finite element analysis; Vibration damping

1. Introduction

Piezoelectric materials such as lead zirconate titanate (PZT) are extensively used in vibration damping applications [1]. For this purpose, piezoelectric patches are attached to a vibrating structure and can be used for active control or semi-passive damping to reduce undesired vibrations. Active control requires high-voltage amplifiers to drive the piezoelectric actuators, and its realization and implementation includes complex setups and high hardware requirements. Furthermore, the closed loop may exhibit instability due to spillover effects [2] of uncontrolled or unmodelled eigenmodes of the system. A combination of active control concepts with viscoelastic passive damping layers [3,4] reduces this destabilizing spillover effects.

In contrast to active control, semi-passive damping concepts require only a simple passive electrical network (PEN) and have the advantage of guaranteed stability. The idea of semi-passive damping enhancement was firstly presented in theory and experiment by Forward [5]. Important theoretical work in the area of passive damping of structural vibrations with piezoelectric materials was further contributed by Hagood and von Flotow [6], Lesieutre

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[7,8], Law et al. [9] and others. A survey of recent modelling approaches of passively damped structures in the context of control engineering can be found in Moheimani [10].

Semi-passive damping may be obtained by bonding piezoelectric elements onto a structure and connecting the electrodes to an external PEN. Due to straining of the host structure, e.g. as a result of transverse vibrations, stresses are generated within the piezoelectric material. As a result of the direct piezoelectric effect, a fraction of the mechanical energy of the vibrating structure is converted into electrical energy. The converted electrical energy is stored as free charge on the surface if the piezoelectric material is in the open-circuit condition. If a PEN is connected to the electrodes, a portion of the accumulated electric energy will be dissipated through the resistive components of the PEN.

The application of models based on the Finite Element Method (FEM) represents an interesting alternative to analytical and semi-analytical formulations. FE formulations discretize continuous systems leading to a finite number of equations. The volume-coupled piezoelectricity is hereby included [11]. An overview of piezoelectric finite element modelling is given in [12,13]. Great emphasis has also been placed on modelling piezoelectric sensors and actuators [14,15]. The main advantage of the FEM is its widespread use in the engineering community. In comparison, algorithms related to an analytically based strain energy approach are cumbersome. Additionally, structures having complex geometry can also be treated. FE-based analysis of semi-passive damped structures can be used as a general design tool, provided that the large number of dynamic equations are sufficiently reduced. Furthermore, this paper shows that parameter and geometry variations can be performed quite easily. Moreover, the stiffening and added-mass effects due to the applied piezoelectric patches can be incorporated in the FE model. Finally, the performance increase by using inductors as part of the PEN is demonstrated.

2. Passive vibration reduction of piezoelectric structures

The three-dimensional piezoelectric constitutive law can be written as [16]

$$\boldsymbol{T} = \boldsymbol{c}^E \boldsymbol{S} - \boldsymbol{e}^{\mathrm{T}} \boldsymbol{E}, \tag{1a}$$

$$\boldsymbol{D} = \boldsymbol{e} \boldsymbol{S} + \boldsymbol{\epsilon}^S \boldsymbol{E}, \tag{1b}$$

where $\boldsymbol{E}$ denotes the electric field vector, $\boldsymbol{T}$ the mechanical stress vector, $\boldsymbol{S}$ the mechanical strain, and $\boldsymbol{D}$ the electric charge vector per unit area. The indirect piezoelectric effect is given by Eq. (1a), whereas Eq. (1b) characterizes the direct piezoelectric effect. The matrix $\boldsymbol{e}$ denotes the piezoelectric stress coupling matrix, $\boldsymbol{c}^{\mathrm{E}}$ the mechanical stiffness matrix at constant electric field, and $\boldsymbol{\epsilon}^{\mathrm{S}}$ the permittivity matrix under constant strain.

For modelling purposes, the shunted piezoelectric element is expressed by an equivalent Thevenin's circuit [9], i.e. a current source in parallel with a capacitor. The capacitance is given as

$$C = \frac{A\epsilon_{33}^T}{h_{\mathrm{p}}}, \tag{2}$$

where A is the area of an electrode, ϵ_{33}^T denotes the permittivity at constant stress, and h_{p} is the thickness of the piezoelectric element as depicted in Fig. 1. The model proposed by Hagood and von Flotow [6] is based on a change in stiffness of the piezoelectric material as a result of the impedance of the applied PEN. This characteristic of a shunted piezoelement is modelled by means of a complex modulus, similar to that of viscoelastic materials. In this study, the external admittance of the PEN is purely resistive. This is because a resistor is the only means of dissipating energy. However, the inclusion of inductive elements might improve the efficiency. The frequency dependent loss-factor η for a resistively shunted piezoelectric element is given in [6] as

$$\eta = \frac{k_{31}^2 \Omega_0}{1 + \Omega_0^2 - k_{31}^2} \quad \text{with } \Omega_0 = RC\left(1 - k_{31}^2\right)\omega, \tag{3}$$

where Ω_0 denotes non-dimensional frequency, R is the shunt-resistance of the PEN, and ω is the driving circular frequency. The parameter k_{31} represents the transverse piezoelectric coupling coefficient. By differentiating Eq. (3) with respect to Ω_0, the maximum loss-factor of a resistively shunted piezoelectric element can be found as [6]

$$\eta^{\max} = \frac{k_{31}^2}{2\sqrt{1 - k_{31}^2}}, \tag{4a}$$

$$\text{at} \quad \Omega_0 = RC\left(1 - k_{31}^2\right)\omega = \sqrt{1 - k_{31}^2}. \tag{4b}$$

It can be seen from Eq. (4a) that the maximum loss-factor can be shifted to any desired frequency ω by tuning the resistance R according to Eq. (4b). For commercially available PZT-material, the loss factor can be as high as 6% for a patch vibrating in the (3, 1)-mode.

Focussing on the discussion above, it becomes evident that the characteristics of a semi-passive PZT-element is similar to that of a lossy material. For the overall structural system of Fig. 1 with N attached PZT-elements, the resulting loss-factor for the rth structural mode is defined as [8,17]

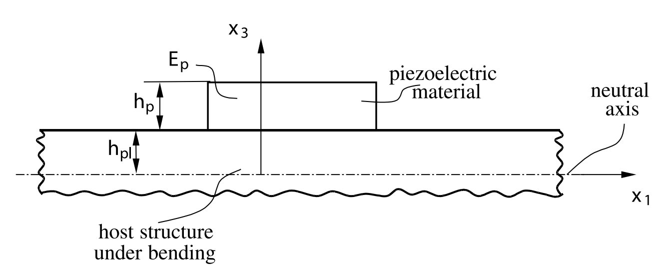

Fig. 1. Flexurally vibrating host structure with attached piezoelement.