



# A hybrid chaos control approach of the performance measure functions for reliability-based design optimization



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## ABSTRACT

Performance measure approach (PMA) is an effective tool of the reliability-based design optimization (RBDO). And the advanced mean value (AMV) method is widely used for the evaluation of probabilistic constraint due to its simplicity and efficiency. However, the AMV method shows instability and inefficiency when applied to the concave performance measure functions, so do other existing iterative methods. In this paper, to overcome the difficulties, a modified chaos control (MCC) is applied to the AMV iterative procedure through modifying the iterative step of the chaotic dynamics analysis. Since the MCC method is inefficient for convex performance measure functions, a hybrid chaos control (HCC) method is also proposed by employing the AMV method or the MCC method adaptively during the RBDO process. Moreover, we equip PMA and sequential optimization and reliability assessment (SORA) with the HCC method for solving RBDO problems. Numerical examples are presented to demonstrate the simplicity, efficiency and robustness of the HCC method.

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## 1. Introduction

In design optimization, it is usually assumed that there is no uncertainty in engineering systems. However, the existence of inherent randomness in physical quantities, such as element dimensions, material properties and external loads, calls for a probabilistic optimization approach. To this end, the reliability-based design optimization (RBDO) has been proposed [1–5]. In general, RBDO methods are divided into three categories [6]: two-level methods, single loop methods and decoupled methods.

Two-level methods solve a RBDO problem using two nested loops: optimization loop (outer loop) and reliability analysis loop (inner loop). In each cycle, the reliability analysis loop is called repeatedly for the deterministic optimization. The reliability analysis loop is a sub-optimization problem, which can be solved by either the reliability index approach (RIA) [1] or the performance measure approach (PMA) [3]. Specifically, RIA applies the first-order reliability method (FORM) [7], which transforms the probabilistic constraint to reliability index constraint for searching the most probable failure point (MPFP). PMA evaluates probabilistic constraint through solving the inverse reliability problem [8,9]. The optimum point of the probabilistic constraint problem on

the target reliability surface is named as the most probable target point (MPTP). When compared with RIA, PMA is more efficient and robust [10–15]. For PMA, the advanced mean value algorithm (AMV) [16] is commonly used to search for MPTP due to its simplicity and efficiency. However, the iterative scheme of AMV suffers from convergence difficulties when concave performance measure functions are involved.

For this reason, several improved iterative procedures, such as conjugate mean value method (CMV) [11,14] and hybrid mean value method (HMV) [11,14], were proposed to guarantee the convergence of AMV. Although both the CMV and HMV perform well for convex and concave performance measure functions, the iterative procedures are still confronted with difficulties on convergence for highly nonlinear performance measure function [12,13]. On this account, other improved algorithms, such as enhanced hybrid mean value method (EHMV) [12,13] and chaos control (CC) method of the performance measure function [15], were proposed. It should be noted that, the CC method performs well for nonlinear performance measure function, but it is computationally inefficient.

Recently, some other advanced RBDO strategies were proposed, such as single loop methods [17–20] and decoupled methods [21–24]. Single loop methods only adopt one loop during the RBDO process, in which the reliability analysis is replaced by the Karush–Kuhn–Tucker (KKT) optimality conditions, thus the RBDO

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### Nomenclature

AMV	advanced mean value	$F_{G_i}(\bullet)$	CDF of the performance measure function
CC	chaos control	$G_i$	the $i$ th performance measure function
CMV	conjugate mean value	$\mathbf{n}$	normalized steepest descent direction of performance measure function
EHMV	enhanced hybrid mean value	$\tilde{\mathbf{n}}$	modified steepest descent direction of performance measure function
FORM	first-order reliability method	$P_f(\bullet)$	failure probability of performance measure function
HCC	hybrid chaos control	$P_i^t$	failure probability
HMV	hybrid mean value	$\mathbf{u}$	independent standard normal random variable
KKT	Karush–Kuhn–Tucker	$\mathbf{u}_{AMV}^k$	the $k$ th standard normal random variable using AMV method
MCC	modified chaos control	$\mathbf{u}_{CMV}^k$	the $k$ th standard normal random variable using CMV method
MCS	Monte Carlo simulation	$\mathbf{u}_{\beta=\beta^t}^*$	the MPFP at target reliability index
MMA	method of moving asymptotes	$\mathbf{x}$	random variable
MPFP	most probable failure point	$\zeta^{k+1}$	criterion for performance measure function
MPTP	most probable target point	$\text{sign}(\zeta^{k+1})$	sign function of $\zeta^{k+1}$
PMA	performance measure approach	$\Phi(\bullet)$	standard normal cumulative distribution function (CDF)
RBDO	reliability-based design optimization	$\beta_i$	reliability index
RIA	reliability index approach	$\beta_i^t$	target reliability index
SORA	sequential optimization and reliability assessment	$\nabla_{\mathbf{u}} G$	derivative of performance measure function $G$
STM	stability transformation method	$\lambda$	chaos control parameter
$C(\mathbf{d})$	objective function	$\delta$	shifting vector of SORA
$\mathbf{d}$	design variable	$\varepsilon$	convergence precision
$\mathbf{d}^L, \mathbf{d}^U$	lower and upper bounds of design variable		
$f(\mathbf{u}^k)$	the next iterative point in standard normal space		
$f_{\mathbf{x}}(\mathbf{x})$	joint probability density function of the random variable		

problems can be solved very efficiently for linear and weakly non-linear problems [6,25]. Decouple methods transform the RBDO to deterministic optimization sequentially. Du and Chen [21] proposed the sequential optimization and reliability assessment (SORA) methods, in which the reliability constraints are substituted by deterministic constraints and the boundaries of constraints are shifted to the feasible domain.

In this paper, a simple and robust new method is proposed to enable an efficient search of MPTP for large-scale application. Our main contributions lie in the following aspects. Firstly, how CMV, HMV and CC methods improve the convergence of AMV method is analyzed in detail. Secondly, a modified chaos control (MCC) method of performance measure function is proposed by extending the CC iterative point to the probabilistic constraint. Thirdly, a function type criterion is applied to distinguish the performance measure function type to reduce the number of function evaluations during the RBDO process. And this is realized by implementing it into PMA and SORA. Finally, several examples are tested to demonstrate the efficiency and robustness of the proposed method.

The outline of this paper is organized as follows. The formulation of two basic RBDO approaches and their difference on reliability analysis is given in Sections 2 and 3, respectively. Then, the detail of MCC method is presented in Section 4. In Section 5, several examples are used to illustrate different MPTP evaluation algorithms for PMA. Subsequently, hybrid chaos control (HCC) method is proposed and tested with PMA and SORA in Section 6. Finally, the conclusion is drawn in Section 7.

## 2. Formulations of reliability-based design optimization

A typical formulation of RBDO model can be depicted as

$$\begin{aligned} \min_{\mathbf{d}} \quad & C(\mathbf{d}) \\ \text{s.t.} \quad & P_f(G_i(\mathbf{d}, \mathbf{x}) \leq 0) \leq P_i^t \quad i = 1, \dots, n_p \\ & \mathbf{d}^L \leq \mathbf{d} \leq \mathbf{d}^U \end{aligned} \quad (1)$$

where  $C(\mathbf{d})$  is the objective function.  $G_i$  is the  $i$ th performance measure function with respect to the design vector  $\mathbf{d} = [d_i]^T$  and the random vector  $\mathbf{x} = [x_i]^T$ .  $P_i^t$  represents the failure probability that is evaluated by the target reliability index function  $\beta_i^t$  as  $P_i^t = \Phi(-\beta_i^t)$ . The failure probability of performance measure function  $P_f(G_i(\mathbf{d}, \mathbf{x}) \leq 0)$  is formulated by the cumulative distribution function  $F_{G_i}(0)$  as

$$P_f(G_i(\mathbf{d}, \mathbf{x}) \leq 0) = F_{G_i}(0, \mathbf{d}) = \int_{G_i(\mathbf{d}, \mathbf{x}) \leq 0} \dots \int f_{\mathbf{x}}(\mathbf{x}) d\mathbf{x} \leq \Phi(-\beta_i^t) \quad (2)$$

where  $f_{\mathbf{x}}(\mathbf{x})$  is the joint probability density function of the random variable vector  $\mathbf{x}$ . The cumulative distribution probability  $F_{G_i}(0, \mathbf{d})$  is the function of performance measure constraint  $G_i(\mathbf{d}, \mathbf{x})$  at the  $i$ th iterative step.

The RBDO probabilistic constraints can be evaluated in two alternative ways

$$\beta_i(\mathbf{d}) = (-\Phi^{-1}(F_{G_i}(0, \mathbf{d}))) \geq \beta_i^t \quad (3)$$

$$G_i(\mathbf{d}) = F_{G_i}^{-1}(\Phi(-\beta_i^t), \mathbf{d}) \geq 0 \quad (4)$$

where  $\beta_i(\mathbf{d})$  and  $G_i(\mathbf{d})$  are the reliability index and the probabilistic performance measure, respectively. When Eq. (3) is employed to describe the probabilistic constraint in Eq. (1) with reliability index, it is the so-called reliability index approach (RIA).

$$\begin{aligned} \min_{\mathbf{d}} \quad & C(\mathbf{d}) \\ \text{s.t.} \quad & \beta_i^t \leq \beta_i(\mathbf{d}, \mathbf{x}) \quad i = 1, \dots, m \\ & \mathbf{d}^L \leq \mathbf{d} \leq \mathbf{d}^U \end{aligned} \quad (5)$$

Similarly, Eq. (4) is applied to describe the probabilistic constraint in Eq. (1) by the performance measure function, which is called the performance measure approach (PMA).

$$\begin{aligned} \min_{\mathbf{d}} \quad & C(\mathbf{d}) \\ \text{s.t.} \quad & G_i \geq 0 \quad i = 1, \dots, m \\ & \mathbf{d}^L \leq \mathbf{d} \leq \mathbf{d}^U \end{aligned} \quad (6)$$

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