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A Lagrangian–Eulerian formulation for reeling analysis of history-dependent multilayered beams

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ABSTRACT

This paper presents a novel Lagrangian–Eulerian finite element formulation for reeling analysis of multilayered beams with gross interlayer slippage. In contrast to the conventional Lagrangian approach, the mesh becomes practically fixed in space, which yields significant benefits for the performance of the contact algorithms and the overall computational efficiency. The needed Lagrangian–Eulerian kinematic relations are derived, special attention is given to the convective transport term for the constitutive variables and an implicit update scheme for the elasto-plastic bending model is formulated. The proposed formulation is shown to predict responses with the same accuracy as offered by the conventional Lagrangian formulation.

1. Introduction

Flexible pipes and cables are widely used in the petroleum industry for fluid transport, power supply and signal transmission. Due to the compliant behavior in bending, these structures are reeled onto large carousels when transported to the production site. A typical reeling operation involves several kilometers of material, large-deformation zones, history-dependent material effects and extensive contact interactions along the reeling path. Use of the conventional finite element (FE) method to simulate such an operation is a very demanding task, however, the process is fairly steady state and the material transport occurs with low velocity along a virtually fixed route in space. This naturally points toward a quasistatic Lagrangian–Eulerian formulation in which the material and the mesh are not tied to each other. In that case the treatment of the history-dependent material behavior becomes the main challenge.

Most of the arbitrary Lagrangian–Eulerian (ALE) procedures for path-dependent materials utilize an operator split method, see e.g. Refs. [1,2]. In such methods the time step is first simplified into a pure Lagrangian step which determines the material motion. Thereafter, the mesh velocity and the updated mesh coordinates are computed in the second step. Finally, a convection step is initiated to transfer the state variables to the new integration points established in the second step. Due to the convection step, the equilibrium is disturbed which leads to some loss of accuracy. However, greater flexibility regarding the mesh motion is allowed for [3], simpler and more robust algorithms can be developed [2] and upgrading of an existing implicit Lagrangian implementation into the ALE description is facilitated since no additional tangent stiffness contributions and global unknowns appear. Successful applications of the fully coupled approach where the mesh motion, the material motion and the convective effects are handled together have also been reported, see Refs. [4,5].

The ALE description has been employed in three-dimensional beam problems to handle joints sliding along the beam axis [6] and to model axial mass flow [7]. Problems involving axially moving beams have been addressed with both the Eulerian description [8] and the Lagrangian–Eulerian approach [9]. In the literature for fluid conveying pipes, the Eulerian formulation is commonly applied, see the review paper by Païdoussis [10] and references cited therein. Common for these applications is the focus on the dynamic behavior and the justifiable assumption of elastic structural responses. To the authors knowledge, the Lagrangian–Eulerian viewpoint has not yet been utilized in beam problems with history-dependent material behavior.

When conventional ALE formulations are used in large-deformation analysis of solids, their objective is to reduce mesh distortions without resorting to re-meshing and updating of the mesh topology. In this work, the purpose is instead to separate the mesh and the material motions such that the mesh becomes practically fixed in space. For reeling analyses this strategy yields significant benefits in terms of,

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(1) Use of non-uniform meshes with large elements in small-deformation regions.
(2) No need to model the vast amount of material stored on the reels.
(3) Improved stability performance for the contact algorithms.
(4) Larger time steps due to less variations in the contact conditions.
(5) Less changes in the contact element topology result in reduced time consumption for contact searches.

Regarding item (1), the overall element size in a conventional Lagrangian simulation is restricted by accuracy requirements at the most critical location along the reeling path. In contrast, a Lagrangian–Eulerian description with a nearly fixed mesh imposes restrictions only for the elements located within the large-deformation regions. This benefit yields, together with item (2) above, a significant improvement of the overall computational efficiency. With regard to item (3), contact interactions often lead to convergence problems in reeling analyses, making frequent user interaction throughout the simulation runs unavoidable. The Lagrangian–Eulerian procedure mitigates these issues as the motion of the contact geometry is significantly reduced. Furthermore, use of low-order contact elements is motivated as continuity of the contact geometry becomes less critical.

Due to the hysteretic bending behavior, significant bending-induced torques may develop in multilayered beams [11]. This phenomenon originates from interlayer friction effects, which allow the bending moment vector to be non-aligned with the normal vector of the curvature plane, such that an internal torque is needed to fulfill equilibrium. In situations with low tension, as is the case in reeling operations, the torque may provoke torsional instabilities with possible formation of loops, see Refs. [12–14]. Indeed, such incidents have been reported by subsea contractors in recent years. To obtain accurate predictions of the generated torque, all history-dependent effects that influence the reeling path configuration must be accounted for, together with the coupling between bending and axial–torsional responses introduced via the interlayer contact pressures.

Several hysteretic bending models have been developed for multilayered beams during the last years. Tan et al. [15] proposed two bending models for flexible risers that were able to account for interlayer friction effects. A more sophisticated approach able to account consistently for radial pressure loads was proposed by Alfano, Bahtui and coworkers [16,17], who employed small-scale detailed FE simulations to calibrate elasto-plastic models for flexible risers. The disadvantage of such an approach is the considerable efforts needed to determine the material parameters. This was remedied by Sævik [18] who formulated two elasto-plastic bending models that were able to account explicitly for the radial interlayer contact pressures. The models were merged into multilayered beam elements and predicted fatigue damages for flexible pipes in good correlation with experiments. Recently, Aguiar et al. [19] proposed a multilayered beam element for flexible pipe analysis, however, explicit account was not made for the interlayer contact pressures.

Based on the background information above, the Lagrangian–Eulerian formulation must comply with the following requirements,

- Predict steady and unsteady responses with the same accuracy as offered by the conventional Lagrangian simulation.
- Properly describe the hysteretic bending behavior and the interaction between bending and axial–torsional responses.
- Provide accurate predictions of the bending-induced torque.

The developed formulation may also be advantageous for simulation of subsea pipeline and cable laying operations. In that case the mesh motion at the upper end is set equal to the motion of the surface vessel. Compared to a conventional Lagrangian simulation, the motion of the pipeline mesh relative to the surface vessel reduces significantly, such that the benefits stated above in item (1)–(5) apply. This approach is regarded as more favorable than the re-use of element strategy proposed by Økland et al. [20].

The organization of the paper is as follows: Section 2 describes the constitutive model employed for the numerical investigations. Section 3 presents the key novelties of the work, in which the kinematic relations, the treatment of convective transport effects for the constitutive variables and the update of the elasto-plastic bending model are addressed. Thereafter, the virtual work contribution is presented and the constitutive part of the tangent stiffness relation is derived. The numerical examples in Section 4 focus on benchmarking against the conventional Lagrangian approach and demonstration of basic performances. A simple reeling example is also presented to illustrate the new computational strategy offered through the developed formulation. In Section 5 the work is summarized and the conclusions are given.

2. Multilayered beam constitutive model

In Section 4 an unbonded flexible pipe is employed for benchmarking, numerical performance tests and to simulate an idealized reeling operation. This pipe represents a typical example of a multilayered beam with gross interlayer slippage. The constitutive model of the pipe is outlined in the following, with emphasis on the history-dependent bending behavior as it represents the main challenge for the Lagrangian–Eulerian formulation. The axial–torsional constitutive relations are also presented due to their coupling with the elasto-plastic bending response.

The pipe cross-section is as illustrated in Fig. 1 made of several concentric layers. With regard to load-bearing, the pressure helix layer supports radial loads, the plastic sheaths contribute mainly in bending and the tensile helix layers balance axial, torsional, bending and radial loads. Unbonded flexible pipes are categorized as slender structures with negligible shear deformations. The shear forces are consequently computed from the moment equilibrium equation, such that a constitutive description must be assigned to the axial force N, the torque M_1 and the bending moments M_2 and M_3. The generalized stress vector $\hat{\boldsymbol{\sigma}}$ and the energy-conjugate strain vector $\hat{\boldsymbol{\epsilon}}$ therefore reads,

$$\hat{\boldsymbol{\sigma}} = \sum_{i=1}^{K} [N_i \quad M_{1i} \quad M_{2i} \quad M_{3i}]^\top \tag{1}$$

$$\hat{\boldsymbol{\epsilon}} = [\epsilon \quad \kappa_1 \quad \kappa_2 \quad \kappa_3]^\top \tag{2}$$

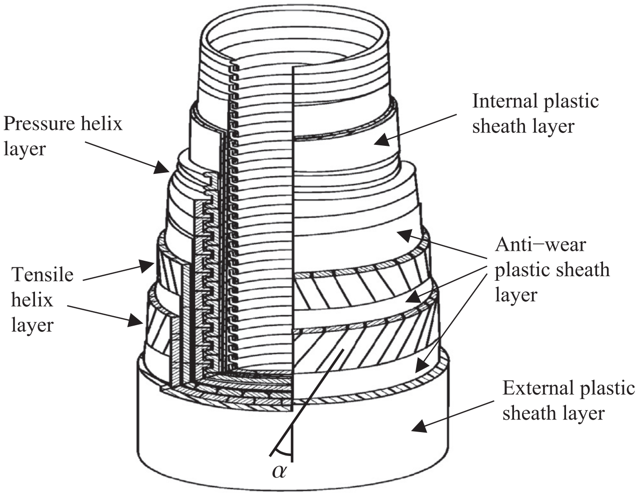

Fig. 1. Unbonded flexible pipe cross-section.