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Large deformation analysis of functionally graded elastoplastic materials via solid tetrahedral finite elements

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ABSTRACT

In this paper, a solid finite element formulation applied to the analysis of functionally graded materials (FGMs) under finite elastoplastic deformation with mixed hardening is presented.

The main novelty of this paper is the use of gradually variable material coefficients in finite elastoplastic strain regime.

The numerical simulations are performed with the same constitutive models but with different variations of the material coefficients. Full integration and high order tetrahedral elements are used. Results show that high order elements and mesh refinement avoids general locking problems. Finally, the differences (regarding final results) between the homogeneous and the FG cases are depicted.

1. Introduction

The aim of this study is to analyze, via solid finite elements, the behavior of functionally graded (FG) elastoplastic materials under finite deformations and strains. These materials have many engineering applications. At the automotive industry level, one can cite the sheet-metal forming processes, in which some automobile components acquire the desired shape via plastic deformation ([1–3]). Functionally graded materials (FGMs) are special composites in which the composition and, thus, the mechanical properties vary gradually (smooth and continuously) over the volume. This special variation avoids the material mismatch, which may cause delamination failures, common in laminated composites. The main applications of these advanced composites are in thermal environments, such as high-speed aircrafts, nuclear reactors and chemical power plants. Besides, highly deformable materials are very common nowadays, such as metals, which can present large displacements, and rubber-like materials, which can withstand large strains. In order to predict the structural behavior of engineering materials, and to reduce costs and time during the design process, scientists and engineers have been using the Finite Element Method (FEM). In this study, the element adopted is the isoparametric solid tetrahedral finite element of any-order (see, for instance, [4,5]). Regarding the numerical results, mesh refinement and full numerical integration are performed in order to obtain an accurate and reliable solution.

The equilibrium analysis of elastoplastic materials under finite deformations is not a simple task. To analyze flexible structural components under large displacements, the geometrically nonlinear analysis is imperative. In addition, to describe the material behavior in the finite elastoplastic strain regime, a general framework has been recently developed. In this framework, called hyperelastoplascity, the finite elastoplastic deformations are described by means of a multiplicative gradient decomposition ([6–10]). In general, for elastoplastic materials under finite strains, the additive decomposition of the strain tensor, called Green-Naghdi decomposition [11], is not valid (see the work of [12] for further details). In this regime, the multiplicative decomposition, called Kröner-Lee decomposition [13,14], is more suitable and well-accepted. Some concepts from the small strain elastoplasticity can be extended to the finite strain regime, such as yield criterion, plastic flow, hardening, consistency condition and return algorithm.

According to [15], FGMs have great potential in many engineering sectors, such as aerospace, automobile and defense industries, as well as electronics and biomedics. Concerning FG elastoplastic materials, some works present in the scientific literature may be cited. Akis and Eraslan [16] have presented plane strain analytical solutions of rotating FG hollow shafts. In [17], spherical pressure vessels composed of a FG elastoplastic material are investigated analytically. A finite element analysis of circular

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Table 1
The expressions describing the plastic model.

Internal dissipation inequality:

$$d = (\mathbf{M}_e - \boldsymbol{\chi}) : \mathbf{D}_p - \left(\frac{\partial \psi_p}{\partial \boldsymbol{\chi}}\right) \cdot \overset{\circ}{\boldsymbol{\chi}} - \left(\frac{\partial \psi_p}{\partial \kappa}\right) \cdot \dot{\kappa} \geqslant 0 \tag{18}$$

Plastic spatial velocity gradient:

$$\mathbf{L}_p = \mathbf{D}_p + \mathbf{W}_p \tag{19}$$

Plastic strain rate tensor:

$$\mathbf{D}_p = \left(\mathbf{F}_p^{-T}\right) \dot{\mathbf{E}}_p \left(\mathbf{F}_p^{-1}\right) = \mathrm{sym}(\mathbf{L}_p) \tag{20}$$

Plastic spin tensor:

$$W_p = \mathrm{ant}(\mathbf{L}_p) \tag{21}$$

Objective Jaumann rate of the intermediate backstress:

$$\overset{\circ}{\boldsymbol{\chi}} = \dot{\boldsymbol{\chi}} - \mathbf{W}_p \boldsymbol{\chi} + \boldsymbol{\chi} \mathbf{W}_p \tag{22}$$

Three-dimensional von-Mises yield criterion:

$$\phi = \phi(\mathbf{M}_e - \boldsymbol{\chi}, \kappa) = \|dev(\mathbf{M}_e - \boldsymbol{\chi})\| - \sqrt{\frac{2}{3}} \sigma_\kappa(\kappa) \leqslant 0 \tag{23}$$

Associative plastic flow rule:

$$\mathbf{D}_p = \dot{\lambda} \mathbf{R}_P = \dot{\lambda} \frac{\partial \phi}{\partial \mathbf{M}_e} = \dot{\lambda} \frac{dev(\mathbf{M}_e - \boldsymbol{\chi})}{\|dev(\mathbf{M}_e - \boldsymbol{\chi})\|} \tag{24}$$

Swift isotropic hardening law:

$$\sigma_\kappa = Y_0 \cdot (\varepsilon_0 + \kappa)^n \tag{25}$$

Isotropic hardening parameter:

$$\dot{\kappa} = \dot{\lambda} r_\kappa = \sqrt{\frac{2}{3}} \|\mathbf{D}_p\| = \sqrt{\frac{2}{3} (\mathbf{D}_p : \mathbf{D}_p)} = \dot{\lambda} \sqrt{\frac{2}{3} (\mathbf{R}_P : \mathbf{R}_P)} = \dot{\lambda} \sqrt{\frac{2}{3}} \tag{26}$$

Nonlinear Armstrong-Frederick hardening law:

$$\overset{\circ}{\boldsymbol{\chi}} = \dot{\lambda} \mathbf{R}_\chi = c \mathbf{D}_p - \dot{\lambda} b \boldsymbol{\chi} \tag{27}$$

Evolution of the initial backstress tensor:

$$\dot{\mathbf{X}} = \dot{\lambda} \mathbf{R}_X = \mathbf{F}_p^{-1} \overset{\circ}{\boldsymbol{\chi}} \mathbf{F}_p^{-T} - sym(2 \mathbf{C}_p^{-1} \dot{\mathbf{E}}_p \mathbf{X}) \tag{28}$$

Null plastic spin tensor:

$$\mathbf{W}_p = \mathbf{O} \Rightarrow \dot{\mathbf{F}}_p = \dot{\lambda} \mathbf{R}_P \mathbf{F}_P, \quad \overset{\circ}{\boldsymbol{\chi}} = \dot{\boldsymbol{\chi}} \tag{29}$$

Consistency condition:

$$\dot{\lambda} \dot{\phi} = 0 \tag{30}$$

Plastic multiplier:

$$\dot{\lambda} = -\frac{\partial \phi / \partial \mathbf{E}}{[(\partial \phi / \partial \mathbf{F}_p) : (\mathbf{R}_P \mathbf{F}_p) + (\partial \phi / \partial \mathbf{X}) : \mathbf{R}_X + (\partial \phi / \partial \kappa) r_\kappa]} : \dot{\mathbf{E}} \tag{31}$$

plates made of FG elastoplastic materials under low-velocity impact is performed in [18]. Rotating disks composed of FG elastoplastic materials are analytically and numerically analyzed in [19]. However, most of (if not all) the related works are restricted to small strain theory. Moreover there is no evidence of convergence analysis regarding mesh refinement in all consulted works related to the subject and, according to the research done by the present authors on the scientific literature, there are no published studies regarding finite element analysis of FG hyperelastoplastic materials. Therefore, another aim of this work is to fulfill the lack of studies related to the numerical analysis of FGMs under finite elastoplastic strains.

This paper is organized in five sections. The exact kinematics and its numerical approximation are described in section two. In the third section, the constitutive framework, the adopted models, the FG laws and the equilibrium principle are given. Fourth section describes some aspects of the incremental procedure performed in order to achieve force equilibrium and plastic consistency. In the fifth section, the numerical examples used to validate the methodology, as well as the discussion of results, are provided. Finally, the main conclusions are given in the sixth section.

2. Kinematics

By kinematics one understands the relation among positions (or displacements) and the strain measure, including its elastic and plastic parts. In this section, the exact kinematics of hyperelastoplasticity and the adopted finite element approximation are described.

2.1. Elastoplastic decomposition

In this study, finite elastoplastic strains are considered. So, the decomposition of the deformation gradient (or change of configuration function) into its elastic and plastic parts is performed here by means of the Kröner-Lee decomposition:

$$\mathbf{F} = \mathbf{F}_e \mathbf{F}_p \tag{1}$$

where $\mathbf{F}$ is the deformation gradient; and the subscripts $()_e$ and $()_p$ denote, respectively, its elastic and the plastic parts. Moreover, it is assumed the existence of an intermediate configuration, which is stress-free and locally defined. From expression (1), one can write:

$$J = \det(\mathbf{F}) = J_e J_p \tag{2}$$

$$\mathbf{E} = \frac{1}{2}(\mathbf{F}^T \mathbf{F} - \mathbf{I}) = \mathbf{F}_p^T \mathbf{E}_e \mathbf{F}_p + \mathbf{E}_p \tag{3}$$

where J is the Jacobian; $\mathbf{E}$ is the Green–Lagrange strain tensor; and $\mathbf{I}$ is the identity matrix.

2.2. Tetrahedral finite element

In the present study, the positional version of FEM is used (see, for instance, [4,5,20–22]). In this case, both initial and current positions, represented here by $\mathbf{x}$ and $\mathbf{y}$, are mapped from a non-dimensional space (ξ):

$$\mathbf{x} = \mathbf{X}^k \phi_k(\xi) \text{ or } x_i = (x_i)^k \phi_k(\xi_m) \tag{4}$$

$$\mathbf{y} = \mathbf{Y}^k \phi_k(\xi) \text{ or } y_i = (y_i)^k \phi_k(\xi_m) \tag{5}$$

where $\mathbf{X}^k$ and $\mathbf{Y}^k$ denote, in this order, the three initial and final coordinates of node k; and ϕ_k is the shape function associated with node k. The deformation gradient (1) is multiplicatively decomposed into two auxiliary gradients: