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An efficient transient analysis method for linear time-varying structures based on multi-level substructuring method

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ABSTRACT

In this paper, the Newmark-beta method for linear time-varying (LTV) structures is firstly derived, then a transient analysis method for LTV structures based on multi-level substructuring method is presented. In theory the proposed method could achieve the same precision as the overall structure analysis method, the computational accuracy is not influenced by substructure partition strategy. The proposed method has higher computational efficiency than overall structure analysis method. Furthermore, the partial substructure tree traversal strategy can further improve the computational efficiency without any loss of accuracy. Some numerical examples are given to illustrate the effectiveness and feasibility of the proposed methods.

1. Introduction

With the rapid development of modern science and technology, the practical engineering structures become increasingly complex and large. Particularly, many of these structures are time-varying structures, which are characterized by (mass, stiffness, damping) properties that change with time, the dynamic problem of time-varying structures is becoming increasing prominent.

At present, for the dynamic problem of time-varying structures extracted from large and complex engineering structures, numerical methods are main ways to investigate dynamic response of time-varying structures [1]. To begin with, we use the finite element method (FEM) for the spatial discretization, which turns the problem into an initial value problem (IVP) of linear ordinary differential equations (ODEs) with time-varying coefficients, and then we use time integration method to solve this ODEs in time. The conventional time integration methods for structural dynamics are primarily focus on solving dynamic response of linear time-invariant (LTI) structures, and the time frozen technique must be adopted when using these methods to solve dynamic response of linear time-varying (LTV) structures, which will bring some errors if time-varying parameters change rapidly [2]. In other words, these methods are far from enough to meet the requirements of engineering precision in many practical engineering problems. So, it is extremely important to develop the transient analysis method for LTV structures.

However, there is few researches on transient analysis method for LTV structures. Penny and Howard [3] developed a time finite element method (TFEM) for time-varying single degree of freedom (SDOF) systems based on Hamilton's principle. Yu et al. [4] developed a TFEM for time-varying multiple degrees of freedom (MDOF) systems based on Hamilton's law of varying action. However, the above two algorithms are only suitable for the case of the mass is gained or lost at null velocity with respect to an inertial reference frame for variable mass systems. For this reason, Zhao and Yu [1] first presented primal form of Hamilton's law of variable mass system, and developed six TFEMs for LTV structures, however, the TFEMs for LTI structures corresponding to these TFEMs for LTV structures are only conditionally stable. Afterward, Zhao and Yu [2] further presented mixed form of Hamilton's law of variable mass system, and developed four discontinuous TFEMs for LTV structures, the TFEMs for LTI structures corresponding to these TFEMs for LTV structures are unconditionally stable, higher-order accuracy and asymptotic annihilation algorithms. Although these TFEMs have the advantage of high accuracy, the computational effort of these TFEMs is very large, which limits their practical engineering application. Even using the time integration methods for LTI structures based on the time frozen technique to perform transient analysis of LTV structures, their computational effort is very large. Therefore, for the transient analysis of large and complex time-varying structures, reducing the computational effort and improving the computational efficiency have become the main concern.

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Dynamic substructure method is a very efficient method to solve dynamic analysis problem of large complex structure, this method can reduce computational effort and required computational resources and improve computational efficiency, which has been widely applied to the dynamic analysis of large complex structures [5–9]. The traditional dynamic substructure method can be categorized into component mode synthesis (CMS) method [10–12] and interface displacement condensation (IDC) method [13–15], where CMS method is more widely used, however, this method has limitations in terms of the computational accuracy and modeling ability of complex model. The multi-level substructuring (MLS) method can better deal with the modeling of a complex model and improve the computational efficiency, thus many investigators has paid much attention to this method. At present, scholars have achieved many fruitful results on the research of eigenvalue analysis method based on the MLS method. Arora and Nguyen [16] developed a method for calculating natural frequencies and mode shapes of large structures with substructures and the subspace iteration. Lu [17] presented a simplified dynamic condensation in multi-substructure systems. Leung [18] extended dynamic substructure method to multi-level (recursive) substructures, and this method can accurately predict more natural frequencies and mode shapes than the number of degrees of freedom (DOFs) retained. Lu et al. [19] presented a subspace iteration method for solving eigenvalue problems of arbitrary multi-level substructure systems. Zhang et al. [20] presented a combined method based on the multi-level substructuring method and the Lanczos method for solving dynamic characteristics of large and complex structures. Bennighof and his colleagues [21–23] first presented the automated multilevel substructuring (AMLS) method, which is an efficient condensation method to determine hundreds to thousands of approximate natural frequencies and mode shapes as well as frequency responses for large and complex structures, which provides the possibility to solve a large number of eigenvalues of large and complex structures and has been widely used [24–26].

However, there is few studies on transient analysis method based on the MLS method. Bathe and Gracewski [32] presented an efficient analysis method of nonlinear dynamic response of large finite element systems based on substructure and mode superposition method, and the method has been implemented in ADINA. Lu et al. [27] presented a mode superposition substructure method with dynamic superelements for forced vibration analysis of LTI multi-level substructure systems. Zhang et al. [28] presented a direct integration method for forced vibration analysis of LTI multi-level substructure systems. These transient analysis methods for LTI structures based MLS method provide a new idea to greatly reduce the computational effort of the transient analysis methods for LTV structures. However, as far as it is known to the author, the transient analysis method for LTV structures based MLS method has not still been presented until now.

In this work, the Newmark-beta method for LTV structures is firstly derived, and then an efficient transient analysis method for LTV structures based MLS method is presented by combining the proposed Newmark-beta method for LTV structures and MLS method. Next, the computational effort of the proposed methods is investigated, the theoretical analysis indicates that the proposed transient analysis method for LTV structures based on MLS method has lower computational cost and higher efficiency than the overall structure analysis method.

The remainder of this paper is organized as follows. In Section 2, the semi-discrete dynamic equation of LTV structures is briefly given, which is usually obtained by the spatial discretization. In Section 3, the Newmark-beta method for LTV structures is firstly derived, then the computational procedure is given. In Section 4, an efficient transient analysis method for LTV structures based on MLS method is presented, in which the two key techniques involved in the proposed method are discussed in detail: (1) Static condensation and data recovery techniques and (2) Substructure tree traversal technique. Then the computational procedure is discussed in detail. In Section 5, the computational effort of the proposed methods is investigated. In Section 6, the effectiveness and feasibility of the proposed methods are verified extensively by numerical examples, including free and forced vibration of LTV SDOF systems that have analytic solutions, forced vibration of cantilever beam with time-varying mass, forced vibration of multi-story frame structure with time-varying stiffness, and forced vibration of two-dimension plane structure with time-varying stiffness. Final conclusions are given in Section 7.

2. Semi-discrete dynamic equation of linear time-varying structures

According to Hamilton's law of variable mass system [1], after the spatial discretization, the obtained semi-discrete dynamic equation of LTV structures can be expressed as follows:

$$\boldsymbol{M}(t)\ddot{\boldsymbol{x}}(t)+\boldsymbol{C}(t)\dot{\boldsymbol{x}}(t)+\boldsymbol{K}(t)\boldsymbol{x}(t)=\boldsymbol{F}(t) \tag{1}$$

$$\boldsymbol{x}(0)=\boldsymbol{x}_0 \tag{2}$$

$$\dot{\boldsymbol{x}}(0)=\dot{\boldsymbol{x}}_0 \tag{3}$$

where $\boldsymbol{M}$, $\boldsymbol{C}$, $\boldsymbol{K}$ are the time-varying mass, damping, and stiffness matrixes of LTV structures, respectively; $\boldsymbol{M}$ is symmetric and positive definite, $\boldsymbol{C}$ and $\boldsymbol{K}$ are symmetric and positive definite or semi-definite; $\boldsymbol{x}, \dot{\boldsymbol{x}}, \ddot{\boldsymbol{x}}$ are the displacement, velocity, and acceleration vector of LTV structures, respectively; $\boldsymbol{F}$ is the load vector acting on the principal structure, including active forces and additional forces caused by mass changing with time; $\boldsymbol{x}_0$ and $\dot{\boldsymbol{x}}_0$ are the displacement and velocity vectors at the initial time. A dot denotes differentiation with respect to time t. If the velocity vector of the expelled (or gained) mass with respect to inertial reference frame $\boldsymbol{V}_o$ is known condition, then

$$\boldsymbol{C}=\boldsymbol{C}_{vis}+\dot{\boldsymbol{M}} \tag{4}$$

$$\boldsymbol{F}=\boldsymbol{F}_{ext}+\dot{\boldsymbol{M}}\boldsymbol{V}_o \tag{5}$$

while if the velocity vector of the expelled (or gained) mass with respect to principal structure fixed reference frame $\boldsymbol{V}_r=\boldsymbol{V}_o-\dot{\boldsymbol{x}}$ is known condition, then

$$\boldsymbol{C}=\boldsymbol{C}_{vis} \tag{6}$$

$$\boldsymbol{F}=\boldsymbol{F}_{ext}+\dot{\boldsymbol{M}}\boldsymbol{V}_r \tag{7}$$

where $\boldsymbol{C}_{vis}$ represents the viscous damping matrix of the principal structure; $\boldsymbol{F}_{ext}$ represents the vector of external active forces.

3. Newmark-beta method for linear time-varying structures

The direct time integration is a step-by-step numerical procedure. Assuming that the total time T is divided into n time steps, then all the discrete time points are $0=t_0<\cdots<t_k<\cdots<t_n=T$, where $t_k=k\Delta t$ $(k=0,1,\ldots,n)$ and time step is equal to $\Delta t=T/n$.

We are interested in obtaining approximated solutions of Eq. (1) by single-step difference method. The Newmark method is the most popular algorithm for numerical solutions of LTI structural dynamic problems. Similarly, the Newmark-beta method for LTV structures are defined by the following relations:

$$\dot{\boldsymbol{x}}_{k+1}=\dot{\boldsymbol{x}}_k+[(1-\gamma)\ddot{\boldsymbol{x}}_k+\gamma\ddot{\boldsymbol{x}}_{k+1}]\Delta t \tag{8}$$

$$\boldsymbol{x}_{k+1}=\boldsymbol{x}_k+\dot{\boldsymbol{x}}_k\Delta t+[(1-2\beta)\ddot{\boldsymbol{x}}_k+2\beta\ddot{\boldsymbol{x}}_{k+1}]\frac{\Delta t^2}{2} \tag{9}$$

$$\boldsymbol{M}_{k+1}\ddot{\boldsymbol{x}}_{k+1}+\boldsymbol{C}_{k+1}\dot{\boldsymbol{x}}_{k+1}+\boldsymbol{K}_{k+1}\boldsymbol{x}_{k+1}=\boldsymbol{F}_{k+1} \tag{10}$$