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Structural damage identification based on power spectral density sensitivity analysis of dynamic responses

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ABSTRACT

A new method is proposed to identify locations and severities of structural damages based on the power spectral density sensitivity analysis. Firstly, the structural responses and power spectral density under stationary and random excitations are calculated using pseudo excitation method. Then, the sensitivities of power spectral density with respect to the structural damage parameters are obtained. Finally, the finite element model updating method is adopted to identify the structural damages from the calculated and the simulated measured power spectral density. Two numerical examples demonstrate the satisfactory identification results obtained from the present method.

1. Introduction

Efficient methods to detect and quantify structural damage have drawn wide attention from various engineering fields. The state-of-the-art in control and health monitoring in civil engineering structures has been reviewed by Housner et al. [1]. Zou et al. [2] offers an extensive review of the progress on structural condition monitoring and damage identification for composite structures. Doebling et al. [3] provided a comprehensive summary on the damage detection methods by examining changes in the dynamic characteristics of the structure. Recently, dos Santos et al. [4] surveyed damage identification methods which consider the numerical data and the experimental data of the undamaged and damaged structure dynamic characteristics. Fan and Qiao [5] reviewed and compared damage identification methods for beam- or plate-type structures based on different damage identification algorithms.

Damage detection usually requires a mathematical model on the structure in conjunction with experimental model parameters of the structure. The identification approaches are mainly based on the change in the natural frequencies [6,7], mode shapes [8–10], measured modal flexibility [11–13], frequency response function [14,15] or the combination of these methods [16–18]. Limitations of the frequency-domain damage detection methods lie in: (1) natural frequencies of structure are not sensitive to local damages; (2) mode shapes of higher orders cannot be easily measured and the accuracy is low due to limited measurement points.

Among all of the structural damage detection techniques, approaches based on dynamic responses have been a hot research topic during the past 20 years. There are a lot of non-destructive methods in the literatures for structural damage detection in time domain. Time histories of vibration response of the structure were used to identify damage in smart structures [19]. Lu and Law [20] proposed a structural damage identification approach based on response sensitivity analysis in time domain. Ding et al. [21] developed a damage identification procedure based on energy variations of responses decomposed using wavelet packet transform. Damage identification in time domain takes advantage of plenty measure data and can yield satisfactory results; however, the same excitation force for the intact and damaged structures to obtain the structural dynamic responses is required.

Taking advantage of the plenty of response data in frequency domain, Liberatore and Carman [22] proposed an approach for damage identification by analyzing the power spectral density of the structure. The relative changes between input and output energies in specific bandwidths are regarded as the occurrence of damage. And the damage is located by a damage location function, summing all the mode shapes which are weighted by the percentage change caused by the damage. Bayissa and Haritos [23] used spectral strain energy (SSE) analysis to identify structural damage in the context of a non-model-based damage identification approach. The SSE is derived from moment–curvature response in which all the modal parameters are needed. Fang and Perera

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[24] utilized power mode shapes for early damage detection in linear structures. Two damage indices based on the concept of power mode shapes were proposed in the localization of structural damage, and the statistical properties of random signals and bandwidth-localized energy concept were used in the formulation. Kanazawa and Hirata [25] presented a new cross-spectral analysis procedure for the parametric estimation. More recently, Wolfsteiner and Breuer [26] developed a technique to assess fatigue of vibrating rail vehicle under non-Gaussian random excitations.

For many damage identification methods having been developed, certainty loads [20,21] are required in order to compare the dynamic responses of real structure and that of the simulated model; therefore, this would be impractical for large-scale civil engineering structures. Damage identification methods based on structural responses under random excitations such as earthquakes or ambient white noise excitations will provide a possible way to solve the problem.

In this paper, an approach for structural damage identification based on the response power spectral density sensitivity analysis is proposed. The dynamic response of structures and sensitivity of power spectral density with respect to the damage parameters are obtained using stationary, random excitation with pseudo-excitation method (PEM) which is high-efficiency for calculating power spectral density of structural dynamic responses [27–29]. In the process of damage identification, the damage parameters are obtained iteratively using the finite element model updating method [30–32]. Two numerical examples, a plane frame structure and a 12-story shear building structure illustrate the correctness of the proposed method.

2. Methodology

2.1. PEM for random vibration analysis

The (PEM) transforms a stationary random vibration analysis into a series of harmonic response analyses and turns a non-stationary random vibration analysis into a series of transient direct dynamic analyses in the time domain, and so it reduces computation efforts considerably while retaining the theoretical accuracy.

2.1.1. Structure subjected to a single-point stationary random excitation

When a linear system is subjected to a single-point stationary random excitation $f(t)$ with auto-power spectrum $S_{ff}(\omega)$, the auto-power spectrum $S_{xx}(\omega)$ of the response is expressed as

$$S_{xx} = |H|^2 S_{ff}, \tag{1}$$

where H is the frequency response function.

It is easy to see if the excitation $e^{i\omega t}$ is multiplied by a constant $\sqrt{S_{ff}}$ to construct a pseudo excitation, i.e. $\tilde{f}(t) = \sqrt{S_{ff}}e^{i\omega t}$, the response of the structure should be multiplied by the same constant

$$\tilde{x} = \sqrt{S_{ff}} H e^{i\omega t}. \tag{2}$$

Premultiplication conjugation of $\tilde{x}$ for Eq. (2), one has

$$\tilde{x}^* \tilde{x} = |\tilde{x}|^2 = |H|^2 S_{ff} = S_{xx}, \tag{3}$$

where the superscript '*' denotes conjugation.

2.1.2. Structure subjected to multi-point coherent stationary random excitations

When structures are subjected to multi-point coherent stationary random excitations, such as earthquake excitations, random wind loads, these can be regarded as the generalized problem of single-point excitation. It can be easily solved following the method below. The stationary response of linear system is expressed as

$$\boldsymbol{S}_{xx} = \boldsymbol{H}^* \boldsymbol{S}_{ff} \boldsymbol{H}^T, \tag{4}$$

where $\boldsymbol{S_{ff}}$ is the spectral matrix for known excitation force, $\boldsymbol{H}$ is the transfer function matrix, $\boldsymbol{S_{xx}}$ is the response spectral matrix to be solved, the superscripts '*' and 'T' denote conjugation and transpose of a matrix, respectively.

The spectral matrix $\boldsymbol{S_{ff}}$ can be decomposed in the Cholesky scheme as

$$S_{ff} = \boldsymbol{L}^* \boldsymbol{D} \boldsymbol{L}^T = \sum_{k=1}^{m} S_{kk} \boldsymbol{a}_k^* \boldsymbol{a}_k^T, \tag{5}$$

in which $\boldsymbol{L}$ is the lower triangular matrix, $\boldsymbol{D}$ is the diagonal element. $\boldsymbol{a_k}$ is the kth column of $\boldsymbol{L}$, and S_{kk} is the kth diagonal element of $\boldsymbol{D}$.

Assuming that the excitations are fully coherent for the simplicity, m in Eq. (5) is taken as one, and Eq. (5) could be rewritten as

$$\boldsymbol{S}_{ff} = \boldsymbol{a}^* \boldsymbol{a}^{\boldsymbol{T}} S_0. \tag{6}$$

where S_0 is a constant obtained from Eq. (5).

Once the pseudo harmonic excitation is constructed as

$$\tilde{\boldsymbol{f}} = \boldsymbol{a} e^{i\omega t} \sqrt{S_0}. \tag{7}$$

The harmonic response of the system can be expressed as

$$\boldsymbol{x} = \boldsymbol{b} e^{i\omega t}, \tag{8}$$

where $\boldsymbol{b} = \boldsymbol{Ha}\sqrt{S_0}$. The response spectral matrix $\boldsymbol{S_{xx}}$ can be written as

$$\boldsymbol{S}_{xx} = \boldsymbol{x}^* \boldsymbol{x}^T = \boldsymbol{b}^* \boldsymbol{b}^T. \tag{9}$$

2.2. Damage identification based on power spectral density sensitivity analysis

Equation of motion of forced vibration for multiple degrees-of-freedom system is written as

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{C}\dot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} = \boldsymbol{F}(t), \tag{10}$$

where $\boldsymbol{M}$, $\boldsymbol{K}$ and $\boldsymbol{C}$ are system mass, stiffness and damping matrices, respectively. Rayleigh damping theory is adopted, i.e.

$$\boldsymbol{C} = a_1 \boldsymbol{M} + a_2 \boldsymbol{K}, \tag{11}$$

where a_1 and a_2 are two constants, and they are determined from two different modal frequencies ω_i, ω_j and modal damping ratios ξ_i, ξ_j, with the expression of $a_1 = \frac{2\omega_j\omega_i(\omega_j\xi_i - \omega_i\xi_j)}{\omega_j^2 - \omega_i^2}$, $a_2 = \frac{2(\omega_j\xi_j - \omega_i\xi_i)}{\omega_j^2 - \omega_i^2}$.

2.3. Sensitivity of power spectral density with respect to damage parameters

Assume the structure is subjected to a stationary random excitation and it can be decomposed as shown in Eq. (5). Applying the pseudo excitation $\tilde{\boldsymbol{f}} = \boldsymbol{a}e^{\boldsymbol{i\omega t}}\sqrt{\boldsymbol{S_0}}$ to the structure, Eq. (10) can be expressed as

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{C}\dot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} = \tilde{\boldsymbol{f}}. \tag{12}$$

The displacement, velocity and acceleration responses of the structures are written as

$$\boldsymbol{x} = \boldsymbol{Ha} e^{i\omega t} \sqrt{S_0}, \tag{13}$$

$$\dot{\boldsymbol{x}} = i\omega \boldsymbol{Ha} e^{i\omega t} \sqrt{S_0}, \tag{14}$$

$$\ddot{\boldsymbol{x}} = -\omega^2 \boldsymbol{Ha} e^{i\omega t} \sqrt{S_0}. \tag{15}$$