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A new interval uncertain optimization method for structures using Chebyshev surrogate models

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ABSTRACT

This paper proposes a new non-probabilistic interval uncertain optimization methodology for structures. The uncertain design problem is often formulated as a double-loop optimization. Interval arithmetic is introduced to directly evaluate the bounds of interval functions and eliminate the inner loop optimization. A high-order Taylor inclusion function is proposed to compress the overestimation of interval arithmetic. A Chebyshev surrogate model is proposed to approximate the high-order coefficients of the inclusion function. A metaheuristic optimization algorithm is combined with the mathematical programming to search the global optimum. Two numerical examples are used to demonstrate the effectiveness of this method.

1. Introduction

Design optimization of structures has experienced considerable development over the past two decades with a wide range of engineering applications. However, the majority of these works are focused on the investigation of the deterministic problems. In engineering, there are many uncertain factors inevitably related to material properties, geometry dimensions, loads and tolerance in the whole life cycle of design, manufacturing, service and aging of the structure [1], due to inherent uncertainties of real-world systems. As a result, the performance of a structure, such as the robustness and reliability, is always subject to some degree of variations due to various uncertainties. An extreme value of the optimization problem, obtained by traditional (deterministic) approaches, can be simply considered as a maximum attainable value from the point of view of its practical realization. However, the design under the deterministic assumption may not satisfy the expected goal or even lies in the unfeasible region. Hence, there is an increasing demand to consider uncertainties quantitatively in the optimization of structures, to enhance structural safety and avoid failure in extreme working conditions due to the unavoidable variability. To incorporate uncertainties in the design optimization, the deterministic design problem should be suitably modified and enhanced.

The reliability-based optimization (RBO) [2] and the robust design optimization (RDO) [3,4] are two major methods to implement the uncertainty optimization. Du et al. [5] also proposed an integrated framework for design optimization problems under uncertainty, which took both the robustness of the design objective function and the probability of the constraints into account. In traditional RDO and RBO methods, uncertain parameters are mostly treated as random variables. The probability distributions are predefined based on the complete information. However, it is time-consuming and even impossible to achieve complete information to determine precise probability distributions, due to the complexity of engineering problems [6,7]. Furthermore, Ben-Haim and Elishakoff [6] have shown that even small variations deviating from real values may cause relatively large errors to the probability distributions in the feasible region. Hence, the probabilistic methods may experience difficulty for real-world problems. Recently, some non-probabilistic methods have emerged as beneficial supplements to the conventional probabilistic methods.

In engineering, there are a large number of design problems that have uncertain-but-bounded parameters. The uncertainties induced by the bounded parameters can be treated with convex models or interval models [6,8–10]. In particular, the interval model has attracted much attention recently in the optimization of structures [11–14]. In interval models, the interval number is used to measure the uncertainty, because the representation of intervals only requires bounds of uncertain parameters. The determination of lower and upper bounds of an interval is relatively easier, compared to a precise probability distribution. The corresponding

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bounds of an interval function are the minimal and maximal responses of the uncertain objective and constraints. Interval method has been successfully applied to a range of engineering design problems with uncertain-but-bounded parameters [15,16]. Interval models often involve a nested double loop optimization procedure, such as [17,18,19], which are commonly time-consuming, because each function evaluation in the outer optimization is implemented iteratively by an inner optimization.

To reduce the computational cost of the nested optimization, one option is to use the first-order Taylor series expansion to approximate the maximum and minimum values in the inner loop [18], instead of using the optimization algorithm. Chakraborty et al. [20] applied the matrix perturbation theory via a first order Taylor series expansion to obtain a conservative dynamic response of interval functions. Chen et al. [21] used the first-order Taylor series expansion to analyse robust response of interval vibration control systems. In fact, the linearization model optimization is actually a type of degenerative nested optimization, in which the inner optimization is replaced by the first-order Taylor series expansion. However, the linearization model with respect to the first-order Taylor approximation has a lower numerical accuracy, which may lead to a solution located in local unfeasible regions.

In this study, the interval arithmetic [22,23], which defines the fundamental arithmetic operators, is introduced into the inner loop to evaluate the maximum and minimum values of the interval functions, as the interval arithmetic can easily obtain the bounds of a design function with interval variables. However, it is well-known that the range of an interval function is mostly overestimated in the numerical implementation, due to the inherent wrapping effect of the interval arithmetic [22,24].

In the area of structure uncertain analysis, Muhanna et al. [25,26] proposed an element-by-element technique to control the overestimation, which gave the structure response a sharp enclosure. The hybrid method that combines the optimization and interval arithmetic was proposed in [27–29] for the analysis of frequency of structures, to control the overestimation of the response function more effectively. The Taylor inclusion function method [30,31] were proposed for more general problems, which employed the high-order Taylor series to approximate the original function as a polynomial function, and then the interval arithmetic was used to calculate the range of the polynomial function. However, the coefficients, a set of high-order derivatives, in the polynomial function is hard to be obtained even for some functions with explicit expressions. To this end, the Chebyshev series [32] are used to approximate these coefficients of the Taylor inclusion (polynomial) function, so as to develop a Chebyshev surrogate model. This model can be constructed by evaluating function values at specific interpolation points rather than the high-order derivatives, to improve computational efficiency [33,34]. After obtaining the Chebyshev surrogate model for the Taylor inclusion function, the interval arithmetic can be used to directly calculate the bounds of the inclusion function in the inner loop, without requiring the inner loop optimization. There have been several surrogate models developed for the design problems of structures, e.g. the linear model [35], Kriging model [36], artificial neural network [37,38] and support vector machine [39]. However, the aim of the Chebyshev surrogate model in this study is only to approximate the high-order coefficients in the Taylor inclusion function, and other surrogate models are hard to transform to the format of Taylor inclusion functions. The approximated inclusion function will be used to compress the overestimation in the interval arithmetic.

The outer loop optimization mainly aims to update the mean values of the design variables. To search the global optimum in the outer loop optimization, the metaheuristic optimization methods [40–47] can be employed. In this paper, the Multi-Island Genetic Algorithm (MIGA) [48,49] will be combined with the Sequential Quadratic Programming (SQP) [50] in a sequential manner to improve the efficiency, which is easily to implement. That is, the MIGA will be used to find the optimal solution as the initial point of the SQP.

2. Design optimization under interval uncertainties

This section will propose a new uncertainty optimization model, in which both the design variables and structural parameters are considered as interval numbers. In engineering problems, there are many cases that the design variables are also uncertain variables, besides other uncertain parameters. For example, the stiffness in vehicle suspensions can be both the design variable and the uncertain parameters due to the material property variations. The cross section areas of a truss structure can be design variables, and at the same time uncertain parameters due to manufacturing tolerance. When both design variables and other parameters are under uncertainty, the proposed uncertainty optimization model will be more suitable for practical problems. As mentioned above, the information of interval variables can be easily obtained compared to the precise probabilistic distributions of random variables. Specially, the uncertainty of both the objective and constraints induced by interval numbers are calculated, in a way similar to the concept of traditional RDO and RBO, respectively.

A general deterministic optimization model for the design of structures is given by

$$\begin{cases} \min\limits_{\mathbf{x}} & f(\mathbf{x},\mathbf{y}) \\ \text{s.t.} & g_i(\mathbf{x},\mathbf{y}) \leqslant 0, \quad i=1,2,\ldots,n \\ & \mathbf{x}^l \leqslant \mathbf{x} \leqslant \mathbf{x}^u \end{cases} \tag{1}$$

The above mathematical model is used to minimize the objective f subject to constraints g_i. $\mathbf{x} \in R^k$ is the vector including deterministic design variables, and $\mathbf{y} \in R^q$ is the vector of consisting of deterministic parameters. To describe uncertainties in the design, interval numbers are introduced to express the variations induced by the uncertainty. Any interval $[x]$ can be expressed as

$$[x] = [\underline{x}, \bar{x}] = x_c + [\Delta x] \tag{2}$$

where $\underline{x}$ and $\bar{x}$ denotes the lower and upper bounds of $[x]$, respectively, $x_c = (\bar{x} + \underline{x})/2$ denotes the midpoint of $[x]$, and $[\Delta x]$ denotes the symmetric interval of $[x]$, which is defined by

$$[\Delta x] = [-w([x]), w([x])], \quad \text{where} \quad w([x]) = (\bar{x} - \underline{x})/2 \tag{3}$$

where the width $w([x])$ reflects the uncertain degree of $[\mathbf{x}]$.

Considering the uncertainties, the deterministic optimization model (1) can be re-defined as follows:

$$\begin{cases} \min\limits_{[\mathbf{x}]} & f([\mathbf{x}],[\mathbf{y}]) \\ \text{s.t.} & g_i([\mathbf{x}],[\mathbf{y}]) \leqslant 0, \quad i=1,2,\ldots,n \\ & \mathbf{x}^l \leqslant [\mathbf{x}] \leqslant \mathbf{x}^u \end{cases} \tag{4}$$

Here, the ranges for the interval parameters $[\mathbf{y}]$ will in general be pre-determined. Since the width of an interval design variable $[\mathbf{x}]$ is also pre-given as ξ, any interval design variable can be expressed as

$$[\mathbf{x}] = \mathbf{x}_c + [-\xi, \xi] \tag{5}$$

The responses of the objective and constraints would also be interval numbers, denoted by $[f]$ and $[g]$, respectively, because the design variables and parameters are interval vectors.

The above minimization problem is to minimize both the average value and the width of the uncertain objective function, to ensure the "robustness" of the design. The minimization of the width will lead to the decrease of the variance of the objective function, to make the uncertain objective function insensitive to