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Identification of design parameter variability of honeycomb sandwich beams from a study of limited available experimental dynamic structural response data

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ABSTRACT

The goal of this paper is to characterise the variability of two important design parameters of thermoplastic honeycomb sandwich beams from the analysis of experimentally determined resonance frequencies and mode shapes of a limited number of test beams, under free boundary conditions. The design parameters are the Young's modulus of the skin in length direction of the beam and the out-of-plane shear modulus of the honeycomb core. These two independent parameters are expressed as random fields using a Karhunen–Loève series expansion of the covariance matrix, available after updating the finite element models using the experimental vibration data. The random variables of this series expansion are expressed in terms of a Hermite polynomial chaos. The aleatory uncertainty is modelled by the Gaussian random variables while the epistemic uncertainty is described by the randomness of the polynomial chaos coefficients. An estimation of the uncertainty resulting from the experimental and numerical modal analysis is discussed, along with its influence on the two considered stiffness parameters.

1. Introduction

Honeycomb panels are geometrically complex but nominally regular structures. Such panels consist of a honeycomb core that is bonded to thin face sheets. The structure of a typical panel is shown in Fig. 1. Their dynamic behaviour, quantified by resonance frequencies and mode shapes, depends on a large number of parameters related to the beam geometry and the elastic properties of the materials used. This paper considers narrow slices of a sandwich panel which are considered to be beam structures. It studies the beam model parameter variability from its experimental structural behaviour. The beams are cut from Monopan® sandwich [1] panels. Although there is much repetition and regularity in the geometry of such a panel and in the assembly process, it turns out that local material and geometry characteristics exhibit a high degree of scatter. Variability exists at two different levels: from one panel to another (inter-variability) and also between different positions in one specific panel (intra-variability or spatial variability). The objective of this research is to quantify scatter based on measurements on commercially available panels, and to derive a relation between the levels of scatter that are observed on the one hand and the expected physical and mechanical properties on the other hand. The random field method is used as tool for the variability analysis of the considered stiffness parameters.

The elastic mechanical properties of a typical honeycomb core are described and analytically calculated by Gibson and Ashby [2]. They propose formulas for calculation of the in-plane and out-of-plane elastic moduli and Poisson ratios of the core. The main work on the dynamics of sandwich panels is related to conventional foam – core structures. Nilsson and Nilsson [3] tried to analytically predict natural frequencies of a honeycomb sandwich plate with free boundary conditions using Blevins [4] formula in which the mass per square metre and equivalent bending stiffness are frequency dependent. Another, more practical way to predict natural frequencies and mode shapes of a honeycomb panel is by means of finite element (FE) analysis. In the past few years, different new approaches have been developed which incorporate high order shear deformation of the core. Work in this area has been carried out by Topdar [5] and Liu [6–8]. The latter stated that the

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Abbreviations: PC, polynomial chaos; KL, Karhoenen–Loève; FEM, finite element modelling; FEA, finite element analysis; EMA, experimental modal analysis; RF, random field; PDF, probability density function; FRF, frequency response function; FFT, fast Fourier transform; MC, Monte Carlo; MCMC, Monte Carlo Markov chain; BI, Bayesian inference; SVS, shell volume shell; MAC, modal assurance criterion; CI, confidence interval; COV, coefficient of variation.

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Nomenclature

E_s	Young's modulus of the homogenized honeycomb beam skin in length direction
G_c	out-plane-shear modulus of the homogenized honeycomb core material
ρ_c	Mass density of the homogenized honeycomb core material
ε_r	normalized Random Error
H	transfer function
σ	standard deviation
N_a	number of averages for frequency response function measurements
γ^2	squared coherence value
M	number of test samples
N_{exp}	number of points for realizations of the random field
λ_k	k-th eigenvalue of the covariance matrix of experimental realizations
φ_k	k-th eigenvector of the covariance matrix of experimental realizations
δ_{kl}	Kronecker delta
$\eta^{(k)}$	vector with random coefficients of the k-th KL term
μ	truncation number the KL series expansion
$\bar{F}^{\mu}$	discretized random field truncated after μ terms
$\nu,\ \nu \geqslant \mu$	number of independent Gaussian random variables

shear moduli of the core are important factors in the determination of the values of the natural frequencies and the sequence of mode shapes, especially at high frequencies. At low frequencies, natural frequencies are mostly determined by the bending stiffness of the beam. As frequency increases, the core shear stiffness becomes more and more important.

The use of vibration measurement data for the identification of elastic material properties is studied by Lauwagie [9]. His work discusses how Young's moduli, shear moduli and Poisson ratios of laminated materials can be obtained from modal data such as resonance frequencies and mode shapes.

The analysis of variability can be done in various ways. In any statistical analysis the issue of gathering and obtaining a sufficient large set of data is essential [32]. In this study, a population of 22 specimens is used as a basis for statistical analysis. A survey of uncertainty treatment in finite element analysis (FEA) is given by Moens [10]. The focus of the work presented in this article is to make optimal use of the statistical information available from the limited – size experimental data. According to Schuëller [11–14], processes and system behaviour can be regarded as stochastic processes of which the outcome is governed by a set of stochastic random variables. Consequently, this research considers the dynamic behaviour of honeycomb sandwich beams as a stochastic process. In this area a recent approach for quantification of variability describes the quantity or process of interest as a stochastic random field (RF). The random field theory is extensively studied and further developed by Ghanem [15,16]. Soize [17], Desceliers [18], Arnst [19] and Perrin [20] implemented and adapted this theory for inverse problems and for cases where only limited experimental data is available. Mehrez [21,22] adopted this method to describe the variability of the Young's modulus of composite beams from experimental frequency response functions, by also solving an inverse problem.

This paper presents a strategy to identify the variability of structural parameters of honeycomb sandwich beams. These beams are cut from Monopan® sandwich panels. The core of this type of sandwich panels consists of cylindrical polypropylene tubes that are arranged in a dense stacking (with each tube having 6 neighbour tubes at 60° positions, see Fig. 15), and that are welded together. After the tubes are stacked and welded, slices are cut before the face skins are attached. The core is welded to the polypropylene skin by fusion welding using a welding foil. The skin consists of a symmetric glass fibre twill weave with a polypropylene matrix and a nominal thickness of approximately 0.7 mm. A polypropylene finishing foil is welded to the skin outer surface to make it smooth and flat. A set of 22 nominally identical Monopan® beams are used for this study, each having a length of 850 mm, a width of 50 mm and a thickness of 25 mm.

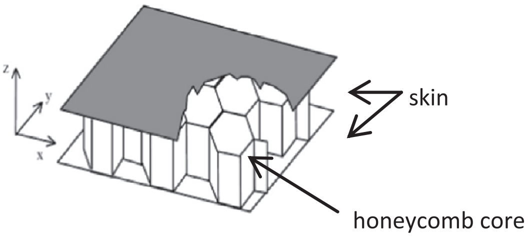

Fig. 1. A typical honeycomb sandwich panel.

The objective of this research is to quantify scatter on the sandwich material properties using experiments and finite element models. The identification procedure consists of five successive steps, most of which are well known already. However, because each step inevitably involves some degree of uncertainty and/or scatter, each step in the procedure is briefly described, with particular attention to the identification of non-determinism that is relevant at that step. The subsequent discussion covers the steps in the logical order:

1. Experimental modal analysis (EMA): the modal parameters of interest here are resonance frequencies and mode shapes of honeycomb sandwich beams with free boundary conditions. The problem of measurement uncertainty and modal parameter estimation uncertainty is also addressed. It is outlined how this EMA uncertainty may be estimated.
2. Finite element modelling (FEM), complemented with a finite element updating procedure of each individual beam: this model is used to create a database with values of two important design parameters of the considered honeycomb beams. In this research the skin Young's modulus in length direction (E_s) and the out-of-plane shear modulus of the honeycomb core (G_c) are studied. These parameters determine the bending stiffness of a beam section.
3. Estimation of errors due to the EMA and FEM processes on the variability of E_s and G_c: the corresponding variability is compared to the estimated physical variability of the elastic parameters.
4. Random field description of the stiffness characteristics of the beam: the probability density functions are estimated for the two parameters at the measurement positions, along with their confidence intervals (CI). The confidence intervals quantify the epistemic uncertainty which arises from the lack of sufficient statistical data. The implementation of the random field model is validated with respect to the number of test samples available.
5. Discussion of the physical relevance of the obtained results: the relation between the estimated PDF's and the real physical variability of the two design variables is discussed. The so-called aleatory uncertainty includes physically related variability,