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Finite element modeling of nonlinear vibration behavior of piezo-integrated structures

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ABSTRACT

This paper aims at finite element modeling of nonlinear vibration behavior of piezo-integrated structures subjected to weak electric field. This nonlinear vibration behavior was observed in the form of dependence of resonance frequency on the vibration amplitude and nonlinear relationship between excitation voltage and vibration amplitude. The equations of motion for the finite element model is derived by introducing nonlinear constitutive relations of piezoceramics in Hamilton's principle. Modal reduction is used to reduce the equations of motion. Thus obtained reduced equation of motion is solved by numerical integration. Experimental validation of the finite element model is also carried out.

1. Introduction

The application of vibration characteristics of piezoceramics have been widespread. So, it is very vital to understand their vibration behavior. The nonlinear vibration behavior of piezoceramic at strong electric field (above 100 V/mm) is a well known phenomenon and has been studied by many researchers. But the nonlinear vibration behavior at weak electric field has not been addressed adequately in the literature. Holland and Eer Nisse [1], during admittance measurement of piezoceramic, observed that if the piezoceramic are excited close to their resonant frequency, they can have nonlinear vibration behavior at weak electric field. Anderson and Crawley [2] observed the nonlinear relationship between electric field strength and strain. They examined the effect of additional mechanical strain on the piezoelectric coefficient d_{31}. Parashar and von Wagner [3] and Parashar et al. [4] found these nonlinear effects in longitudinal vibrations and shear vibrations of free (un-bonded) piezoceramics.

Piezoceramics are seldom utilized in isolation i.e. without bonding them to the structures. Hence, it is desirable to observe the vibration behavior of a piezoceramic integrated structure, such as a cantilever beam. This yields very important information about these structures, which can be utilized for more efficient design. Nonlinear behavior of piezo-beam system subjected to weak electric field is described by Anderson and Crawley [2]. Nguyen [5] and von Wagner and Hagedorn [6] also described and attributed this behavior to nonlinear material property of piezoceramics.

Experiment conducted with the resonant frequency excitation of piezo-beam system at weak electric field showed the nonlinear vibration behavior. The piezo-beam system used in the experiment is shown in the Fig. 1. The piezoceramics are bonded at the middle of the cantilever beam. Piezoceramics are excited by electrical signal close to the second natural frequency of the piezo-beam system. The nonlinear second resonant frequency for the system at the given excitation voltage is measured. It is the frequency which provides maximum response at the given excitation voltage. The response is measured with the help of a laser vibrometer. The plot obtained by such measurement is shown in the Fig. 2. The plot is between the response measured and the frequency ratio. The piezo-beam system shows here a typical softening characteristic of the nonlinear system.

More often the complexity of nonlinear constitutive relation due to the coupling of electrical and mechanical terms precludes the analytical solution for all but the simplest case. It is extremely challenging to obtain the analytical solution even for a moderately complex piezoceramic model. So a numerical solution scheme based on the finite element method is desired. The approximate method such as the finite element method is capable of involving the complex nonlinear material laws with relative ease.

Since the first representative paper on application of the finite element method for piezoelectric by Allik and Hughes [7] a lot of work has been published. Tzou and Tseng [8] developed a "thin" piezoelectric solid element with internal degrees of freedom to

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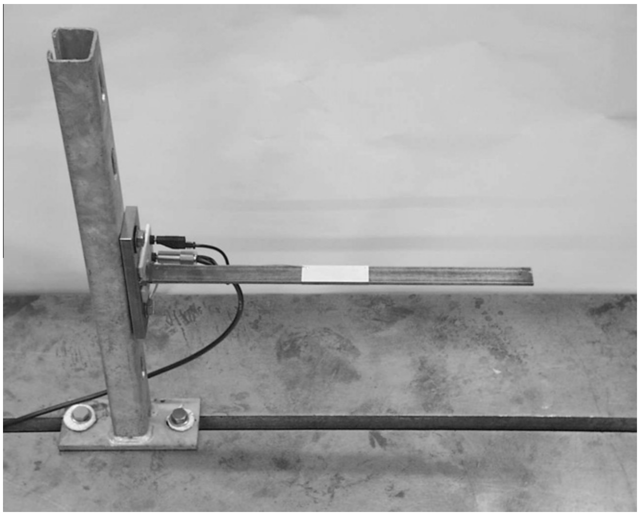

Fig. 1. Piezo-beam system.

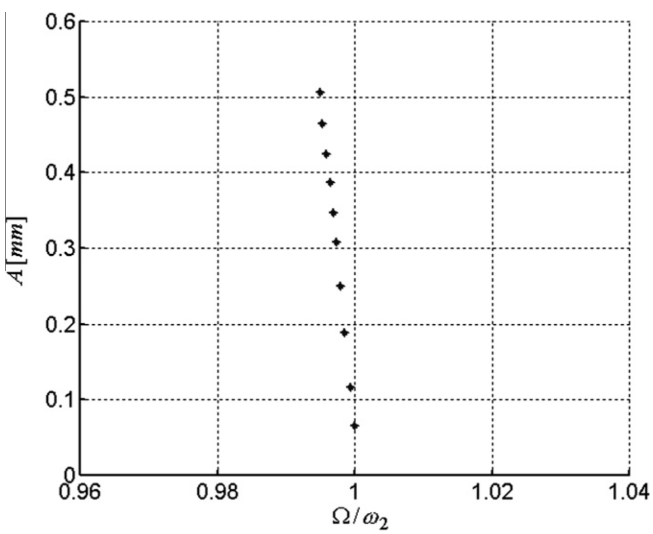

Fig. 2. Back-bone curve (experimental) at second resonant frequency and at $x = 370$ mm.

model the "intelligent plate". They also used the Guyan reduction scheme to condense the degrees of freedom associated with the electrical potential. Peng et al. [9] did finite element modeling of the piezoceramic-composite structure. They used third order laminate theory to model the composite beam containing piezoceramics and then used the mode superposition technique and the Newmark-β method to calculate the dynamic response. Hwang and Park [10] demonstrated the use of four-node, 12 degrees of freedom quadrilateral plate bending element with one electrical degree of freedom. They also used the Hamilton's principle to derive the equations of motion. Similar approach is used in the work of Chen et al. [11] and Liu et al. [12].

All the papers discussed above deal with the linear finite element modeling of piezoceramics. Even though nonlinear behavior of piezoceramic is well known observed phenomenon, nonlinear finite element modeling has not been dealt much in the papers. This has been stressed in the survey paper of Rao and Sunar [13]. Gaudenzi and Bathe [14] gave a generalized iterative finite element procedure in which they have briefly discussed its application in the case of nonlinear constitutive relations. Fung et al. [15] applied the finite element method to predict the nonlinear vibration of a piezoelectric beam contacting with a fixed disk. But in their work they assumed nonlinearity due to frictional contact of the beam with the fixed disk while the nonlinearity due to piezoceramic was not considered. In the work of Samal et al. [16], present authors have modelled the nonlinearity of un-bonded piezoceramics, but the piezoceramic integrated structures have not been dealt before. Besides the complete three-dimensional model used in there gives a large number of nonlinear elastic and damping coefficients which are having no actual practical significance on the behavior of the system.

The present paper provides a finite element model which incorporates the nonlinear constitutive laws of the piezoceramics to predict the nonlinear vibration behavior of piezoceramic integrated structures. Thus obtained finite element model is validated in the light of the experimental results.

2. Finite element model

2.1. Description of the physical model

The piezo-beam system is shown in Fig. 3. The typical dimensions of the beam are; length $l_b = 400$ mm, width $b = 30$ mm and thickness $h_b = 3$ mm. Two piezoceramics are attached to the beam at the middle, one on each side. The dimensions of piezoceramics are; length $l_c = 70$ mm, width $b = 30$ mm and thickness $h_c = 1$ mm. The cantilever beam is attached to a column on one side. Direction of polarization of both the piezoceramics is same. When the piezoceramics are excited by alternating current, it causes expansion in one piezoceramic while contraction occurs in the other piezoceramic at the same time. Due to alternating current electric field reverses its direction in each cycle, which induces bending vibration in the beam.

2.2. Constitutive equations

The length dimension of the beam and the piezoceramic is much higher in comparison to the other two dimensions viz. the width and the thickness. Moreover, the piezoceramics used in the present work have the electric field in the z-direction while the expansion and contraction primarily occurs in the x-direction (d_{31}-effect). Therefore, in the present case the normal stress components in transverse directions T_{yy} and T_{zz} along with the normal strain components S_{yy} and S_{zz} are assumed to be neglected [3]. Similarly, the shear stress components T_{xy}, T_{yz}, T_{zx} are also assumed to be zero. As the electric fields is applied in the z-direction, the electric fields E_x and E_y in the directions opposite to the direction of applied electric field are neglected. Utilizing notation of IEEE standards [17] the constitutive equations in the present case reduce to

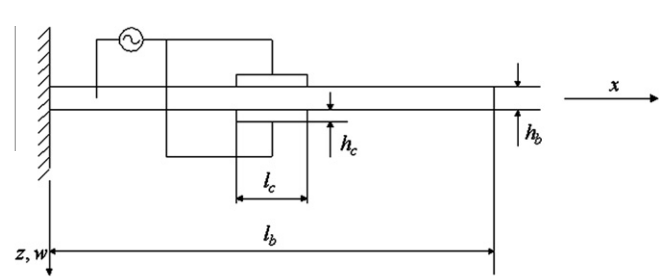

Fig. 3. Physical model of the piezo-beam system.