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MGARCH models: Trade-off between feasibility and flexibility

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ABSTRACT

Multivariate GARCH (MGARCH) models need to be restricted so that their estimation is feasible in large systems and so that the covariance stationarity and positive definiteness of conditional covariance matrices are guaranteed. This paper analyzes the limitations of some of the popular restricted parametric MGARCH models that are often used to represent the dynamics observed in real systems of financial returns. These limitations are illustrated using simulated data generated by general VECH models of different dimensions in which volatilities and correlations are interrelated. We show that the restrictions imposed by the BEKK model are very unrealistic, generating potentially misleading forecasts of conditional correlations. On the other hand, models based on the DCC specification provide appropriate forecasts. Alternative estimators of the parameters are important in order to simplify the computations, and do not have implications for the estimates of conditional correlations. The implications of the restrictions imposed by the different specifications of MGARCH models considered are illustrated by forecasting the volatilities and correlations of a five-dimensional system of exchange rate returns.

1. Introduction

It is common for conditional variances, covariances and correlations of multivariate time series to be required in practice. For example, the majority of fund managers use volatility and conditional correlation forecasts in the context of systems of financial returns, for constructing equity portfolios and pricing assets and for risk management and future hedging procedures; see Amenc, Goltz, Tang, and Vaidyanathan (2012) for a survey based on 139 North American Investment managers. In addition to financial applications, conditionally heteroscedastic multivariate models have also been fitted to systems of macroeconomic and commodity related variables; see for example Conrad and Karanasos (2010) for the inflation-growth interaction and Bampinas and Panagiotidis (2015) for the relationship between oil and gold prices. Finally, multivariate models of the conditional variances and correlations are also useful when modeling and forecasting non-economic time series; see for example Cripps and Dunsmuir (2003) and Jeon and Taylor (2012), who model the wind speed and direction. In this context, multivariate GARCH (MGARCH) models are very popular and have attracted a great deal of attention; see Bauwens, Laurent, and Rombouts (2006), Engle (2009), Francq and Zakoïan (2010, Ch. 11) and Silvennoinen and Teräsvirta (2009) for surveys.

The original MGARCH model, proposed by Bollerslev, Engle, and Wooldridge (1988) and denoted by VECH, is quite flexible, allowing all volatilities and conditional covariances to be interrelated; see Table 1 for a summary of the acronyms of all of the models and estimators considered in this paper. However, the empirical implementation of VECH models is limited due to their extremely large numbers of parameters, even in systems of moderate dimensions. An additional problem that VECH models face is that of determining the conditions required to guarantee positive definite conditional covariance matrices and/or stationarity. These problems are overcome by restricting

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Table 1
Acronyms for multivariate GARCH (MGARCH) models and estimators (inalphabetical order).

Acronym	Model/estimator	Authors
Acronyms for models		
AS-DCC	Almon-shuffle DCC	Bauwens, Grigoryeva, and Ortega (2016)
BEKK	Baba-Engle-Kraft-Kroner	Engle and Kroner (1995)
CCC	Constant conditional correlation	Bollerslev (1990)
cDCC	Corrected DCC	Aielli (2013)
DBEKK	Diagonal BEKK	Ding and Engle (2001)
DCC	Dynamic conditional correlation	Engle (2002)
D-RBEKK	Diagonal RBEKK	Noureldin, Shephard, and Sheppard (2014)
DVECH	Diagonal VECH	Bollerslev et al. (1988)
ECCC	Extended CCC	Jeantheau (1998)
EcDCC	Extended cDCC	
EDCC	Extended DCC	Engle (2002)
GDCC	Generalized DCC	Hafner and Franses (2009)
GO-GARCH	Generalized OGARCH	Boswijk and van der Weide (2011)
O-GARCH	Orthogonal GARCH	Alexander (2001)
RBEKK	Rotated BEKK	Noureldin et al. (2014)
RDCC	Rotated DCC	Noureldin et al. (2014)
SBEKK	Scalar BEKK	Ding and Engle (2001)
S-RBEKK	Scalar RBEKK	Noureldin et al. (2014)
VECH	VECH	Bollerslev et al. (1988)
Acronyms for estimators		
EBE	Equation-by-equation	Francq and Zakoïan (2016)
G-CVT	Gaussian Covariance targeting	Engle and Mezrich (1996)
G-QML	Gaussian quasi-maximum likelihood	Bollerslev and Wooldridge (1992)
LSW	Ledoit-Santa Clara-Wolf	Ledoit et al. (2003)
S-CVT	Student covariance targeting	
S-QML	Student quasi-maximum likelihood	Fiorentini, Sentana, and Calzolari (2003)

the MGARCH models that are often fitted in practice in such a way that parameter estimation is feasible and/or covariance stationarity and positiveness are guaranteed. Some popular restricted VECH models are the diagonal VECH (DVECH), suggested by Bollerslev et al. (1988), and the BEKK (Baba-Engle-Kraft-Kroner) model of Engle and Kroner (1995); see for example Bauwens, Hafner, and Rombouts (2007) and Ledoit, Santa-Clara, and Wolf (2003) for applications of DVECH models, and Beirne, Caporale, Schulze-Ghattas, and Spagnolo (2013), Hafner and Herwartz (2008b) and Hecq, Laurent, and Palm (2016) for the BEKK model.

Alternatively, several specifications of restricted MGARCH models are based on the decomposition of the covariance matrix into the product of conditional variances and correlations. The most popular of these models are the Constant Conditional Correlation (CCC) model of Bollerslev (1990) and the Dynamic Conditional Correlation (DCC) model of Engle (2002); see for example Amado and Teräsvirta (2014) and Laurent, Rombouts, and Violante (2013) for some recent empirical applications of the CCC model, and Aielli and Caporin (2014), Audrino (2014), Bauwens, Hafner, and Pierret (2013) and Engle and Kelly (2012) for the DCC model. However, if the restrictions behind these models are not satisfied, the estimated conditional variances, covariances and correlations may suffer from biases. Furthermore, given that different models are based on different restrictions, the choice of a particular restricted MGARCH model can lead to substantially different conclusions when forecasting dynamic covariance matrices; see Kroner and Ng (1998) and Ledoit et al. (2003), who compare the DVECH, BEKK and CCC models empirically and show that they may have different economic implications. As a consequence, it is important to analyze the effects of the restrictions imposed by the different MGARCH specifications that are usually fitted in practice for the prediction of conditional variances, covariances and correlations.[1] This is the main objective of this paper. We carry out Monte Carlo experiments by fitting several popular restricted MGARCH models to systems that are simulated by specifications without restrictions and analyze the empirical implications of the restrictions imposed on the models for the prediction of conditional variances, covariances and correlations.

Our work is related to the empirical comparisons carried out by Caporin and McAleer (2008, 2012), who focus on the scalar versions of the BEKK and DCC models. It is also related to those of Caporin and McAleer (2014) and Rossi and Spazzini (2010), who carry out Monte Carlo experiments for comparing MGARCH models, focusing respectively on the interactions between the specification of the conditional covariance matrix and the conditional distribution of returns and on the effect of the dimension of the system. We extend previous research by including new models and very recently proposed new parameter estimators that have not been considered in previous comparisons. Furthermore, our focus is on the biases that are introduced by the restrictions when predicting conditional variances, covariances and correlations. Our results have important implications when MGARCH models are used to predict volatilities and conditional correlations.

The rest of this paper is organized as follows. Section 2 describes the MGARCH specifications considered,

[1] Caporin and McAleer (2012) argue that the particular specification that is fitted to the data in empirical applications is often chosen on an *ad hoc* basis, with the ease of estimation being a primary factor in the selection of the model.