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Forecasting multidimensional tail risk at short and long horizons

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ABSTRACT

We define the Multidimensional Value at Risk (MVaR) as a natural generalization of VaR. This generalization makes a number of important applications possible. For example, many techniques developed for VaR can be applied to MVaR directly. As an illustration, we employ VaR forecasting and evaluation techniques. One of our forecasting models builds on the progress made in the volatility literature and decomposes MVaR into long-term trend and short-term cycle components. We compute short- and long-term MVaR forecasts for several multidimensional time series and discuss their (un)conditional accuracy.

1. Introduction

The interest in multidimensional tail (MT) events is driven by their importance in economics, finance, insurance and many other areas of applied probability, statistics and decision theory. The modeling and forecasting of MT events is paramount for many important applications in economics and finance, such as portfolio decisions (e.g., Ang and Bekaert, 2002), risk management (e.g., Embrechts, McNeil, and Straumann, 2002; Meine, Supper, and Weiß, 2016), multidimensional options (e.g., Cherubini and Luciano, 2002), credit derivatives, collateralised debt obligations and insurance (e.g., Hull and White, 2006; Kalemanova, Schmid, and Werner, 2007; Su and Spindler, 2013), contagion, spillovers and economic crises (Bae, Karolyi, and Stulz, 2003; Hautsch, Schaumburg, and Schienle, 2015; Zheng, Shi, and Zhang, 2012), systemic risk and financial stability (Adrian and Brunnermeier, 2016; Gonzáles-Rivera, 2014), and market integration (e.g., Bartram, Taylor, and Wang, 2006; Lehkonen, 2015).

Tail events are related closely to extreme risk that is generally defined as the potential for significant adverse deviations from the expected results. In the univariate context, a measure of extreme risk that is used widely in practice is the Value at Risk (VaR). VaR is defined as the maximum loss on a portfolio over a certain period of time that can be expected with a nominal probability. However, modern risk management generally involves more than one risk factor and is particularly concerned with the evaluation and balancing of their impacts. For example, multifactor models (e.g., Chen, Roll, and Ross, 1986; Ferson and Harvey, 1998) are used to measure and manage the exposure to each one of multiple economy-wide risk factors.

This paper discusses a new angle on the modeling and forecasting of multidimensional tail events. Building on related recent literature (e.g., Polanski and Stoja, 2012; Prékopa, 2012; Torres, Lillo, and Laniado, 2015), we apply a generalized version of VaR, the Multidimensional Value at Risk (MVaR), which is defined as a value that delimits a multidimensional tail with a nominal probability mass under a given density function. MVaR can be seen as an illustration of the multiple sources of risk: if VaR is a univariate risk measure, which instead of the variance takes into account the entire tail density, then MVaR is a measure of multidimensional risk that instead of the covariances takes into account the entire multidimensional tail density.

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Why should we care about MVaR when in typical portfolio applications it is the portfolio VaR that matters and not the multidimensional tail risk of the components of the portfolio? Although VaR might be the appropriate risk measure in portfolio applications, MVaR is useful in other circumstances where either the risk sources cannot be aggregated to form an informative risk measure or the portfolio interpretation of a collection of variables is not natural, useful or possible.

A prominent example of the importance of accounting for the distributional characteristics of the multiple sources of risk properly comes from the stress testing of portfolios or financial systems. Stress testing frameworks typically begin by developing scenarios with negative outlooks (tail events) for the evolution of certain economic drivers (e.g., GDP growth, interest rates, unemployment, stock market performance, investor sentiment), then proceed to evaluate the impacts of these on portfolios or systemically important institutions (e.g., Bank of England, 2015; European Banking Authority, 2016). Treating with these drivers individually presents a problem, as they are obviously interdependent. Moreover, it would be difficult to construct a portfolio of these factors and use its VaR as a tail risk measure. For example, what is the appropriate weight and its interpretation for each source of risk in such a portfolio? The only alternative is to consider the sources of risk jointly. In this case, MVaR can simplify the task considerably.

Another example related to stress testing that highlights the importance of MVaR is systemic risk. This is the risk of collapse that is faced by the financial system as a whole when one of its constituent parts gets into financial distress. The interconnectivity of financial institutions means that a shock faced by one institution in the form of a tail event increases the probability of other financial institutions experiencing similar tail events, leading to a domino effect (e.g., Gai and Kapadia, 2010; Hautsch, Schaumburg, and Schienle, 2014; Rogers and Veraart, 2013). In this case, it would be both inappropriate and uninformative to treat the financial system as a portfolio of banks and compute its VaR.

Therefore, while it is important to have a measure of the aggregate tail risk, often it is also important to know the direct dependence on, interrelationships among as well as the co-dynamics of the specific sources of tail risk. By focusing on the joint distribution of the individual sources of tail risks, we provide a framework for characterizing the co-dependence of these risks.

One important advantage of MVaR is that, in principle, any techniques and applications that have been developed for VaR can also be applied directly to MVaR. We illustrate this here with both short- and long-term MVaR forecasting and evaluation. First, we obtain one-step-ahead MVaR forecasts using the conditional autoregressive value at risk (CAViaR) of Engle and Manganelli (2004). However, CAViaR is a purely statistical model and does not distinguish between long-term, persistent movements in the tails, driven perhaps by macroeconomic and company fundamentals, and transitory movements that are due to investor sentiment or other short-lived effects. With this in mind, we investigate a new two-factor forecasting model that we apply to MVaR. This model has several advantages. It is simple to estimate and can easily produce multi-step-ahead forecasts. Our two-factor model (2FM) decomposes MVaR into a long-term trend and a short-term cycle which can then be examined for relationships with economic and other variables. Finally, we use the scaling property of financial and economic time series to forecast MVaR at different frequencies and horizons. We evaluate the MVaR forecasts by employing adapted conditional and unconditional VaR forecast evaluation techniques. To the best of our knowledge, this paper is the first to raise these issues in relation to (multidimensional) tail events.

2. Multidimensional value at risk

For the continuous and strictly increasing CDF F (PDF f) of a unidimensional random variable Y on the real line, the VaR at the nominal level a is usually identified with the quantile q_a for which $F(q_a) = a$. More generally, VaR can be defined as the cutoff q_a such that the probability mass under f of the interval $\{y \in R : y/d \geq q_a\}$ for a non-zero number d is equal to a. Depending on the value of d, this definition can apply to either the left ($d < 0$) or right ($d > 0$) tail of a distribution, and also allows for normalization.

In analogy to VaR, for a joint CDF F (PDF f) of a vector $\boldsymbol{Y} = (Y_1, \ldots, Y_N)$ of N random variables on R^N with continuous and strictly increasing marginal CDFs, the Multidimensional Value at Risk (MVaR) in direction $\boldsymbol{d} \in R^N$ at the nominal probability level a is the unique cut-off value $q_a^{\boldsymbol{d}} \in R$, such that the set

$$\mathcal{M}_a^{\boldsymbol{d}} = \left\{\boldsymbol{y} \in R^N : y_i/d_i \geq q_a^{\boldsymbol{d}},\ \forall d_i \neq 0\right\} \quad (1)$$

has probability mass a under f. We refer to the set $\mathcal{M}_a^{\boldsymbol{d}}$ as the MVaR-region or multidimensional tail. In Fig. 1, we illustrate the construction of the multidimensional tail $\mathcal{M}_a^{\boldsymbol{d}}$ as a Cartesian product of univariate tails (VaR-intervals)

$$\mathcal{M}_a^{\boldsymbol{d}} = \left\{y_1 \in R : y_1/d_1 \geq q_a^{\boldsymbol{d}}\right\} \times \cdots \times \left\{y_N \in R : y_N/d_N \geq q_a^{\boldsymbol{d}}\right\},$$

where the probability mass for each VaR-interval $\left\{y_i \in R : y_i/d_i \geq q_a^{\boldsymbol{d}}\right\}$ can be computed from the corresponding marginal CDF.

We also say that $\mathbf{x} \in R^N$ is an extreme observation when $\mathbf{x}$ lies in the MVaR-region. The directional vector $\boldsymbol{d}$ (together with the significance level a) defines the region of interest, and also has a distinct financial interpretation. For example, in the case of systemic risk, the choice of the directional vector hinges on the particular economic metric that is of interest to the regulator. This could be, for example, how much the regulator may have to 'pour into' an institution that is in distress in order to prevent it from 'infecting' its counterparties, where Core Equity Tier 1 (CET1) capital, as one of the most important macroprudential policy ratios for financial stability, is an obvious candidate. If a bank gets into distress and 'eats up' its CET1 ratio, the regulator may be forced to bail it out by providing funding equal to CET1 to return the bank's capital to its pre-distress level. Suppose that a financial system is made up of three banks with CET1 ratios of 2, 1 and 4. Then,