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Symmetric matrix-valued frequency to time transformation for unbounded domains applied to infinite beams

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Abstract

In structural dynamics coupled systems with unbounded deformable members are characterized by radiation damping. Typically, the unbounded subsystem is described in the frequency domain; either numerically or analytically by means of dynamical stiffness matrices. Recent papers describe a matrix-valued rational interpolation of the dynamical stiffness and straightforward transformation into the time-domain. In addition, the asymptotic behaviour has been considered, too, by adding fractional derivatives. However, the matrices involved in this process are unsymmetric even if the original dynamical stiffnesses are symmetric. The approach presented in this paper maintains the symmetry a priori without any numerical operations by simply using a rational approximation with a matrix-valued numerator but a scalar-valued denominator and contains some further numerical advantages. The method is demonstrated by treating an infinite beam on an elastic foundation.

Keywords: Frequency-to-time transformation; Radiation damping; Dirichlet-to-Neumann map; Infinite beam

1. Introduction

There is a wide range of engineering applications where dynamic phenomena within infinite or semi-infinite media are involved. Consider for example scattering or radiation problems in acoustics, surface water waves in oceanography or the problem of finding the stresses and displacements within the earth in geophysics. As an example, Fig. 1(a) shows a typical setup of a soil–structure interaction problem with a turbomachine which causes transients at startup or shutdown. Thus the system will go through the resonance of the supporting structure and any damping, here especially the radiation damping, will become essential. A consistent description of this radiation damping has been tackled by means of different concepts. For transient excitations, nonlinear materials or geometrically irregular setups, a direct finite element discretization may be beneficial. However, disadvantages arise from the truncation of the infinite medium by a bounded grid. Over the last 20 years, strong efforts have been made to develop measures to prevent the reflection of outgoing waves at artificial boundaries, some of them are outlined in [1]. Exact and approximate representations on artificial interfaces have been summarized by Givoli in [2]. Generally speaking, the basic method of this paper, too, is an approximate Dirichlet-to-Neumann transformation on the truncation boundary of the infinite structure.

In a series of papers [3,7,8,9], Ruge et al. proposed a matrix-valued rational relation in the coupling interface between the state variables $\mathbf{z}_c(t)$ (nodal displacements and velocities) and the nodal forces $\mathbf{f}_c$ assuming a harmonic behaviour in the time domain

$$\left.\begin{aligned}\mathbf{f}_c &= \hat{\mathbf{f}}_c \exp(\mathrm{i}\Omega t)\\ \mathbf{z}_c &= \hat{\mathbf{z}}_c \exp(\mathrm{i}\Omega t)\end{aligned}\right\}, \quad \hat{\mathbf{f}}_c = \mathbf{K}(\Omega)\hat{\mathbf{z}}_c, \tag{1}$$

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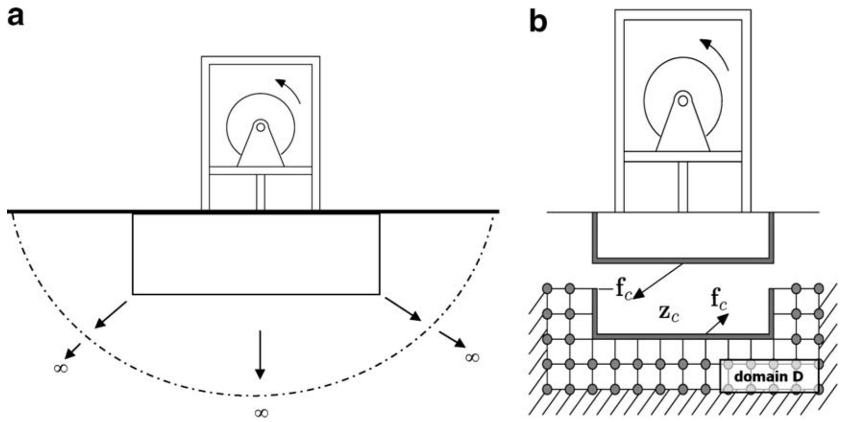

Fig. 1. Soil–structure interaction: (a) system with unbounded soil-domain, (b) system with bounded soil-domain.

either

$$\mathbf{K}(\Omega) = \mathbf{Q}^{-1}(\mathrm{i}\Omega)\mathbf{P}(\mathrm{i}\Omega) + \mathbf{K}_\infty(\mathrm{i}\Omega)$$

or

$$\mathbf{K}(\Omega) = \mathbf{P}(\mathrm{i}\Omega)\mathbf{Q}^{-1}(\mathrm{i}\Omega) + \mathbf{K}_\infty(\mathrm{i}\Omega), \tag{2}$$

where $\mathbf{Q}$ and $\mathbf{P}$ are matrix polynomials:

$$\begin{aligned}
&\mathbf{Q}(\mathrm{i}\Omega) = \mathbf{1} + (\mathrm{i}\Omega)\mathbf{Q}_1 + (\mathrm{i}\Omega)^2\mathbf{Q}_2 + \cdots + (\mathrm{i}\Omega)^M\mathbf{Q}_M,\\
&\mathbf{P}(\mathrm{i}\Omega) = \mathbf{P}_0 + (\mathrm{i}\Omega)\mathbf{P}_1 + (\mathrm{i}\Omega)^2\mathbf{P}_2 + \cdots + (\mathrm{i}\Omega)^{M-1}\mathbf{P}_{M-1},\\
&\mathbf{K}_\infty(\mathrm{i}\Omega) = \lim_{\Omega\to\infty}\mathbf{K}(\Omega), \quad \mathbf{P}_0 = \mathbf{K}(\Omega=0) - \mathbf{K}_\infty(\Omega=0).
\end{aligned} \tag{3}$$

The special choice of $\mathbf{K}_\infty$ and $\mathbf{P}_0$ guarantees the exact asymptotic behaviour at both bounds of the frequency interval.

The unknown matrices $\mathbf{P}_j$ and $\mathbf{Q}_j$ are determined by minimizing the differences between a given set of pairs $(\mathbf{K}_j, \Omega_j)$ and the rational approximation in Eq. (2). However, in order to avoid nonlinear algebraic equations the $\mathbf{Q}$-weighted differences are minimized:

either

$$\sum_{j=1}^{s} |\mathbf{Q}(\mathrm{i}\Omega_j)[\mathbf{K}_j - \mathbf{K}_\infty] - \mathbf{P}(\mathrm{i}\Omega_j)| \to \text{Minimum}$$

or

$$\sum_{j=1}^{s} |[\mathbf{K}_j - \mathbf{K}_\infty]\mathbf{Q}(\mathrm{i}\Omega_j) - \mathbf{P}(\mathrm{i}\Omega_j)| \to \text{Minimum}. \tag{4}$$

The first version in Eq. (4) has been elaborated for a semi-infinite soil-halfspace [3] and for an infinite beam [7] and the results found are really satisfactory. In [7] special attention has been paid to the asymptotic behaviour, which causes fractional derivatives in the time-domain. In [8] the elimination of spurious modes by means of eigenvalue-shifting was added to the frequency-to-time transformation scheme. There is only one property lost when transforming the $\mathbf{K}_j$-set into the rational version; symmetric matrices $\mathbf{K}_j$ do not cause symmetric counterparts $\mathbf{P}_j$, $\mathbf{Q}_j$. Therefore the overall matrix representation of the coupled system will be nonsymmetric even if the pure structure part will be symmetric, a fact which is true in almost all situations in structural dynamics. This unsymmetry does not only cause a double space in the storage memory; solution schemes for symmetric problems are much more efficient and numerically stable. In this paper a truly symmetric version is obtained; the key idea towards a symmetric $\mathbf{P}$, $\mathbf{Q}$ representation comes from modal elimination but for *bounded* domains.

2. Modal elimination

A typical finite-element discretization as shown in Fig. 1(b) for the soil domain D with domain quantities $\mathbf{z}_D$ and interface quantities $\mathbf{z}_C$ results in a matrix representation

$$\begin{bmatrix} \mathbf{A}_{CC} + \mathrm{i}\Omega\mathbf{B}_{CC} & \mathbf{A}_{CD} + \mathrm{i}\Omega\mathbf{B}_{CD} \\ \mathbf{A}_{DC} + \mathrm{i}\Omega\mathbf{B}_{DC} & \mathbf{A}_{DD} + \mathrm{i}\Omega\mathbf{B}_{DD} \end{bmatrix} \begin{pmatrix} \hat{\mathbf{z}}_C \\ \hat{\mathbf{z}}_D \end{pmatrix} = \begin{pmatrix} \hat{\mathbf{f}}_C \\ \mathbf{0} \end{pmatrix} \tag{5}$$

from which the domain-variables can be eliminated:

$$\begin{aligned}
&\Big[(\mathbf{A}_{CC} + \mathrm{i}\Omega\mathbf{B}_{CC}) - (\mathbf{A}_{CD} + \mathrm{i}\Omega\mathbf{B}_{CD})(\mathbf{A}_{DD} + \mathrm{i}\Omega\mathbf{B}_{DD})^{-1}\\
&\quad \times (\mathbf{A}_{DC} + \mathrm{i}\Omega\mathbf{B}_{DC})\Big]\hat{\mathbf{z}}_C = \hat{\mathbf{f}}_C.
\end{aligned} \tag{6}$$

The inversion of a so called λ-matrix ($\lambda = \mathrm{i}\Omega$) can be realized by means of the corresponding right-side modal matrix $\mathbf{X}$ and the left-side modal matrix $\mathbf{Y}$ of the pair $\mathbf{A}_{DD}$, $\mathbf{B}_{DD}$.

$$\begin{aligned}
&(\mathbf{A}_{DD} + \lambda\mathbf{B}_{DD})\mathbf{x} = 0 \to \mathbf{X} = [x_1 \ldots x_{n_D}],\\
&(\mathbf{A}_{DD}^{\mathrm{T}} + \lambda\mathbf{B}_{DD}^{\mathrm{T}})\mathbf{y} = 0 \to \mathbf{Y} = [y_1 \ldots y_{n_D}]
\end{aligned} \tag{7}$$

The well-known orthogonality conditions,

$$\begin{aligned}
&\mathbf{Y}^{\mathrm{T}}\mathbf{A}_{DD}\mathbf{X} = \mathrm{diag}\{\alpha_j\}; \quad j = 1, \ldots, n_D,\\
&\quad \to \mathbf{A}_{DD}^{-1} = \mathbf{X}\,\mathrm{diag}\left\{\frac{1}{\alpha_j}\right\}\mathbf{Y}^{\mathrm{T}},\\
&\mathbf{Y}^{\mathrm{T}}\mathbf{B}_{DD}\mathbf{X} = \mathrm{diag}\{\beta_j\}; \quad j = 1, \ldots, n_D,\\
&\quad \to \mathbf{B}_{DD}^{-1} = \mathbf{X}\,\mathrm{diag}\left\{\frac{1}{\beta_j}\right\}\mathbf{Y}^{\mathrm{T}},
\end{aligned} \tag{8}$$