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# A constraint Jacobian based approach for static analysis of pantograph masts

B.P. Nagaraj [a,1], R. Pandiyan [a,2], Ashitava Ghosal [b,*]

[a] ISRO Satellite Centre, Bangalore 560 017, India
[b] Dept. of Mechanical Engineering, Indian Institute of Science, Bangalore 560 012, India

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ABSTRACT

This paper presents a constraint Jacobian matrix based approach to obtain the stiffness matrix of widely used deployable pantograph masts with scissor-like elements (SLE). The stiffness matrix is obtained in symbolic form and the results obtained agree with those obtained with the force and displacement methods available in literature. Additional advantages of this approach are that the mobility of a mast can be evaluated, redundant links and joints in the mast can be identified and practical masts with revolute joints can be analysed. Simulations for a hexagonal mast and an assembly with four hexagonal masts is presented as illustrations.



## 1. Introduction

Deployable structures can be stored in a compact configuration and are designed to expand into stable structures capable of carrying loads after deployment. In their general form, they are made up of a large number of straight bars (links) connected by revolute joints and with one or more cables used for deployment or increasing the stiffness of the deployed structure (see, for example, [1,2]). Initially, the whole assembly of bars can be stowed in a compact manner and, when required, can be unfolded into a predefined large-span, load bearing structural form by simple actuation of one or more cables. This characteristic feature makes them eminently suitable for a wide spectrum of applications, ranging from temporary structures that can be used for various purpose in ground to the large structures in aerospace industry. Deployable/collapsible mast are often used for space applications since in their collapsed form they can be easily carried as a spacecraft payload and expanded in orbit to a desired size. Many deployable systems use the pantograph mechanism or scissor-like elements (SLE's). Typically, an SLE has a pair of equal length bars connected to each other at an intermediate point with a revolute joint. The joint allows the bars to rotate freely about an axis perpendicular to their common plane. Several SLE's are connected to each other in order to form units which in plan view appear as regular polygons with their sides and radii being the SLE's. Several such polygons, in turn, are linked in appropriate arrangements leading to deployable structures that are either flat or curved in their final deployed configurations. The assembly is a mechanism with one degree of freedom from the stowed/folded configuration till the end of deployment. The deployment is through an active cable and after deployment the assembly is a pre-tensioned structure. Active cables control the deployment and pre-stress the pantograph and passive cables are pre-tensioned in the fully deployed configuration. These cables have the function of increasing the stiffness in the fully deployed configuration [3].

### 1.1. Kinematics and mobility

The kinematics of multi-body mechanical systems can be studied by use of relative coordinates [4], reference point coordinates as used in the commercial software ADAMS [5] or Cartesian coordinates (also called natural/basic coordinates) [6]. In Refs. [7,8], Garcia and co-workers have used Cartesian coordinates to obtain the constraints equations for different types of joints and for kinematic analysis of mechanisms. Typical pantograph masts are over-constrained mechanisms according to Grübler–Kutzbach criteria, and in Ref. [9], Cartesian coordinates have been used to study the kinematics and mobility of deployable pantograph masts – the authors use the derivative of the constraint equations and develop an algorithm to obtain redundant link and joints in over-constrained deployable masts, perform kinematic analysis and obtain global degrees of freedom. The key advantage of Cartesian coordinates is that the constraint equations are *quadratic* (as opposed to transcendental equations for relative coordinates), and, hence their derivatives are *linear*. As shown in [9], these features allows easier

* Corresponding author. Tel.: +91 80 2293 2956; fax: +91 80 2360 0648.
*E-mail address:* asitava@mecheng.iisc.ernet.in (A. Ghosal).
[1] Spacecraft Mechanisms Group.
[2] Flight Dynamics Group.

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manipulation and simplification of expressions in a computer algebra system to obtain symbolic expressions and closed-form solutions for the kinematics of pantograph masts. A disadvantage of Cartesian coordinates is that the number of variables is typically larger and tends to be (on average) in between relative coordinates and reference point coordinates. However, for analysis of pantograph masts, the number is not too large and could be handled without much difficulty in the computer algebra system, *Mathematica*, used in this work.

The masts in their deployed configuration become pre-tensioned structures. For pre-stressed structures with pin jointed bars, the necessary condition for the structure to be statically and kinematically determinate is given by the *Maxwell's rule*

$$3j - b - c = 0 \tag{1}$$

where, $j$ is the number of joints, $b$ is the number of bars or links and $c$ is the number of kinematic constraints. Calladine [10] generalized the *Maxwell's rule* as

$$\begin{aligned} s &= b - r \\ m &= 3j - c - r \\ 3j - b - c &= m - s \end{aligned} \tag{2}$$

where, $m$ is the number of internal mechanisms, $s$ is the number of states of self-stress, and $r$ is the rank of the equilibrium matrix. This equation is referred to as the *extended Maxwell's rule*. The values $m$ and $s$ depends on the number of bars and joints, topology of the connection and on the geometry of the frame work. The numerical values of the vectors describing $s$ and $m$, for a given system, can be determined from the singular value decomposition (SVD) of the equilibrium matrix. The concept of using a Jacobian matrix to evaluate the mobility was first presented by Freudenstein [11] for an over-constrained mechanism. Later, the first and higher order derivatives of constraint equations has been used for under constrained structural systems to evaluate mobility and state of self-stress by Kuznetsov [12,13]

### 1.2. Structural matrix

The mechanism at the end of deployment becomes a pre-tensioned structure and the structural matrices are useful for evaluating the stiffness/displacement of the SLE masts in the deployed configuration. In literature, researchers have used various methods for formulating the structural matrix for an SLE. These are termed as *force method* [14], *displacement method* [15] and *equivalent continuum model* [16]. We describe each of these methods in brief below.

#### 1.2.1. Force method

In the force method, as used by Kwan and Pellegrino [14], the SLE is discretised into four beam elements. The equilibrium, compatability and flexibility matrices are derived for a typical beam element in a local coordinate system using shear force and bending moment relationships. These equations are transformed to the global coordinate system by using the rotation matrices and are assembled for the four beam elements, which make up the SLE. The equilibrium matrix is reduced in size by matrix partitioning and by setting the end moments to zero [18]. In this approach one can evaluate the number of self-stress states and the number of infinitesimal mechanisms of the given system by using singular value decomposition (SVD) of the equilibrium matrix [19].

#### 1.2.2. Displacement method

The displacement method is used by Shan [15] to formulate stiffness matrix for the SLE. In his approach, each link of the SLE is called an *uniplet*. One uniplet of the SLE is modeled as an assembly of two beam elements with mid node at the pivot point of SLE. The stiffness matrix was partitioned to have the translation terms and rotational terms in order. The final reduced stiffness matrix is obtained by condensing and removing the rotational degrees of freedom of all the three nodes. In Ref. [20], the authors have formulated the stiffness matrix for two uniplets, called as a *duplet*, by using the stiffness matrix of the uniplet developed above. Matrix partitioning is used to get the reduced stiffness matrix which condenses the translational degrees of freedom of the pivot node.

#### 1.2.3. Equivalent continuum model

This approach was used to predict the stiffness characteristics of deployable flat slabs when they are subjected to normal loads [16,17]. In this method, the SLE is considered as an *equivalent* uniform beams that deflects identically to the given loading as that of an SLE. The flat large deployable structure is substituted with an equivalent grid of uniform beams running in particular directions The beams are rigidly connected to each other. This arrangement is reduced to an equivalent orthotropic plate of constant thickness and stiffness matrix is obtained. The results predicted by this method are approximate unlike above methods and hence can only be used for initial design phase which reduces the computational time. In an exact finite element modeling the storage space requirements are large for large number of SLE units due to the complicated pivotal connections and hinged connections that require more than one nodal point to be described accurately. The equivalent approach can significantly reduce the computational effort during preliminary design stage.

#### 1.2.4. Comparison of existing methods

The force method gives the additional information about the states of self-stress and infinitesimal mechanisms. The displacement method or equivalent continuum model does not give this information. The force method uses two matrix reductions which reduces the matrix of dimension $18 \times 14$ to $12 \times 8$ in the first step. Further in the second step the matrix dimension is reduced from $12 \times 8$ to $10 \times 6$, to obtain the final reduced equilibrium matrix. The displacement method has a stiffness matrix of dimension $18 \times 18$ for the two assembled beam elements with six degrees of freedom at each node. By condensing the rotational degrees of freedom at all the nodes the matrix dimension reduces to $9 \times 9$. The reduced matrix has only translational degrees of freedom at each node. The equivalent continuum approach is useful for very large repetitive structures. However, this method does not give the accurate results when compared to other two methods and, hence, can be used only for initial design phase to reduce computational time.

As mentioned earlier, at the end of deployment we get a structure capable of bearing loads, and in this paper, we extend the approach in [9] to the static analysis of deployable pantograph masts. We present a new approach to formulate the structural matrices for a typical SLE using Cartesian coordinates, the kinematic equations of the SLE/pantograph element, and the constraint Jacobian matrix. These matrices are derived by using the symbolic computation software Mathematica [21]. The results of formulations obtained by this approach matches exactly with the results of force and displacement based methods. Our approach has the advantages of the force method in evaluating the states of self-stress and infinitesimal mechanisms. However, in our approach, the final reduced equilibrium matrix can be obtained in a single step unlike in the force and displacement methods. In addition, the constraint equations of the links and joints are useful in studying the kinematics behavior of pantograph masts during deployment, in evaluating the redundancy in the links/joints of these over-constrained systems, and in obtaining the final degrees of freedom of the deployable masts. In literature the successive SLE joint connection