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From periodic to chaotic oscillations in composite laminated plates

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Abstract

The geometrically non-linear, linear elastic, oscillations of composite laminated plates are studied in the time domain by direct numeric integration of the equations of motion. A *p*-version finite element, where first-order shear deformation is followed and that was recently proposed for moderately thick plates, is employed to define the mathematical model. By applying transverse harmonic forces, the variation of the oscillations with the angle of the fibres is investigated. With this kind of excitation, only periodic motions with a period equal to the one of the excitation are found. However, introducing in-plane forces, *m*-periodic or quasi-periodic oscillations, as well as chaotic oscillations are computed. The existence of chaos is confirmed by calculating the largest Lyapunov exponent.

Keywords: Laminated plates; *p*-version; Non-linear; Dynamics; Chaos

1. Introduction

Composite laminated plates are used in many areas such as aeronautics, naval industry and microelectronics [1]. If actuated by large excitations at frequencies close to resonance frequencies, those plates may undergo vibrations with large amplitude, therefore in the geometrically non-linear regime. In this case, it has been found that harmonic excitations can produce periodic oscillations where harmonics of the excitation frequency are quite important [2–5].

Quasi-periodic and chaotic oscillations are also possible in composite laminates. In Ref. [6] one composite plate and one composite cylinder are investigated by numerically integrating in the time domain a set of finite element equations. The excitation is provided by a temperature field and the response described by time and phase plots. According to the authors, the highly irregular oscillations of the plate observed after a certain number of cycles – when displacements as large as 15 times the plate thickness are attained – are chaotic due to their irregularity. Moreover, the portrayed motions are quite irregular from the outset, i.e., even for low vibration amplitudes. Maestrello presents in Ref. [7] an experimental analysis on two adjacent aircraft panels forced by a turbulent boundary layer and a pure tone sound. Responses changing from periodic, to quasi-periodic and finally to chaotic – in a characteristic route to chaos [8] – were observed. To describe the type of motion, the time histories, spectral densities and phase portraits were presented and the largest Lyapunov exponent computed for some experimental data. Another noteworthy study is presented in Ref. [9], where the behaviour of a laminated plate under the effects of high-supersonic flow was analysed. The reduced set of equations of motion, which was obtained applying Galerkin's method, was solved by a continuation procedure to determine static solutions; the shooting and multiple-scales methods were employed to find periodic motions and direct numerical integration for non-periodic states.

Accurate numerical prediction of multi-harmonic, quasi-periodic or chaotic oscillations of a structure is difficult to undertake essentially due to two reasons, which

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contribute to large computational time and memory requirements. First, for most cases, an accurate model requires a large number of degrees of freedom, particularly if one takes into account that the non-linear mode shape is not constant and that several modes may influence the response. Second, higher frequencies are likely to appear in the response, therefore a time domain analysis requires the use of small time steps.

A p-version, hierarchical finite element was developed recently following first-order shear deformation theory and applied to study periodic free vibrations [10]. This finite element requires a moderate number of degrees of freedom for accuracy and is not prone to shear locking. It has other known advantages of p-elements [11], like the fact the number of finite elements is determined by the geometry of the structure to study, rather than by precision requirements. Consequently, the user can define a precise computational model without much effort.

In this paper, the p-version element presented in [10] is employed to study forced, i.e., non-autonomous, oscillations of symmetric laminated plates in the non-linear regime. The shear deformation and rotary inertia are considered in the model and the force in the direction transverse to the plate is sinusoidal in time. Direct numeric integration of the equations of motion is carried out by Newmark's method [12]. The Poincaré maps, phase plots and time histories are shown, and the largest Lyapunov exponent of a chaotic oscillation is computed.

2. Equations of motion

The element employed is briefly described in the following paragraphs along with the derivation of the equations of motion. More details on the element can be found in [10].

It is here accepted that the displacement components of a particle along the x and y directions – displacements u, v – and the displacement along the z direction – displacement w – are given by

$$u(x,y,z,t) = u^0(x,y,t) + z\theta_y^0(x,y,t) \tag{1}$$

$$v(x,y,z,t) = v^0(x,y,t) - z\theta_x^0(x,y,t) \tag{2}$$

$$w(x,y,z,t) = w^0(x,y,t) \tag{3}$$

where the superscript "0" represents the middle plane and the independent rotations of the normal to the middle plane about x and y are denoted by θ_x^0 and θ_y^0 (Fig. 1).

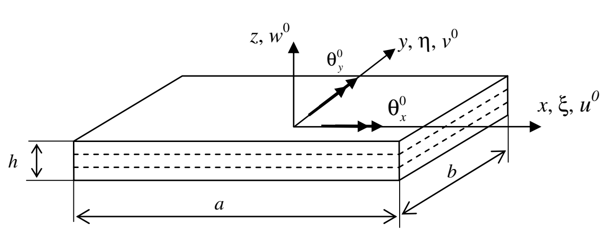

Fig. 1. Plate dimensions, displacements, global and local (ξ and η) coordinates.

The theory resulting from this displacement field is usually designated as the first-order shear deformation theory (FSDT) [1].

For each element, the middle plane displacements and the rotations are expressed as products of space and time functions:

$$\begin{Bmatrix} u^0(x,y,t) \\ v^0(x,y,t) \\ w^0(x,y,t) \\ \theta_y^0(x,y,t) \\ \theta_x^0(x,y,t) \end{Bmatrix} = \begin{bmatrix} \{N^u(x,y)\}^{\mathrm{T}} & 0 & 0 & 0 & 0 \\ 0 & \{N^v(x,y)\}^{\mathrm{T}} & 0 & 0 & 0 \\ 0 & 0 & \{N^w(x,y)\}^{\mathrm{T}} & 0 & 0 \\ 0 & 0 & 0 & \{N^{\theta_y}(x,y)\}^{\mathrm{T}} & 0 \\ 0 & 0 & 0 & 0 & \{N^{\theta_x}(x,y)\}^{\mathrm{T}} \end{bmatrix} \times \begin{Bmatrix} q_u(t) \\ q_{\mathrm{v}}(t) \\ q_w(t) \\ q_{\theta_y}(t) \\ q_{\theta_x}(t) \end{Bmatrix} \tag{4}$$

where $\{q_u(t)\}$, $\{q_{\mathrm{v}}(t)\}$, $\{q_w(t)\}$, $\{q_{\theta_y}(t)\}$ and $\{q_{\theta_x}(t)\}$ are the vectors of generalised displacements and rotations. The complete matrix of shape functions present in (4) is constituted by the row vectors of bi-dimensional in-plane, out-of-plane and rotational shape functions. These row vectors are formed by one-dimensional displacement shape functions as explained in [10]. p_0, p_i, p_{θ_y} and p_{θ_x} are the numbers of transverse, middle plane, rotation about y and rotation about x, one-dimensional shape functions employed.

Concerning the transverse displacements, Legendre polynomials in the Rodrigues' form plus the four Hermite cubics will be used [10]. A set of polynomials called the g set [10] will be applied in conjunction with linear functions for the in-plane displacement as well as for the rotation fields.

The plane stress constitutive equation for the kth layer states that [1]

$$\begin{Bmatrix} \sigma_x^{(k)} \\ \sigma_y^{(k)} \\ \tau_{yz}^{(k)} \\ \tau_{xz}^{(k)} \\ \tau_{xy}^{(k)} \end{Bmatrix} = \begin{bmatrix} C_{11}^{(k)} & C_{12}^{(k)} & 0 & 0 & C_{16}^{(k)} \\ C_{12}^{(k)} & C_{22}^{(k)} & 0 & 0 & C_{26}^{(k)} \\ 0 & 0 & C_{44}^{(k)} & C_{45}^{(k)} & 0 \\ 0 & 0 & C_{54}^{(k)} & C_{55}^{(k)} & 0 \\ C_{16}^{(k)} & C_{26}^{(k)} & 0 & 0 & C_{66}^{(k)} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^{(k)} \\ \varepsilon_y^{(k)} \\ \gamma_{yz}^{(k)} \\ \gamma_{xz}^{(k)} \\ \gamma_{xy}^{(k)} \end{Bmatrix} \tag{5}$$

where $C_{ij}^{(k)}$ are the reduced stiffnesses of the kth layer, which can be obtained from E_1, E_2, major and minor Young's moduli, from the Poisson's ratios ν_{12} and ν_{21} and from the shear modulus G_{12} [1]. The numbers 1 and 2 in the elastic properties, denote the principal directions of the orthotropic plate layer. A shear correction factor, which accounts for the fact that the shear stresses are not constant across the section, is employed [13]. The classic value $\lambda =$