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Structure assembling by stochastic topology optimization

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Abstract

Topology optimization is of significant importance to the design of truss- and grillage-like structures. Conventional topology optimization procedures are usually based on the ground structure approach. Starting from highly connected structures the uneconomical links are eliminated during the course of optimization. In this paper we show that, additionally, stochastic methods offer the possibility to build-up structures starting from simple initial configurations with few elements. Stochastic optimization methods (simulated annealing, evolutionary algorithms, random cost) are applied to the topology design problem on the basis of appropriate local structure variations. Results and performance comparisons are given.

Keywords: Stochastic optimization; Structural optimization; Topology optimization; Weight optimization; Evolution strategy; Simulated annealing

1. Introduction

Attempts to apply optimization algorithms to the complex task of structural design have been made over a considerable time. One of the many problems encountered in this field is the topological design of discrete structures like trusses. The different approaches can roughly be divided into the following categories [8,21]: The most common approach is the ground structure method where the optimization is started from highly connected initial structures. During the optimization process the unnecessary structural members will be eliminated (see e.g., [4]). It is evident that in practical applications the ground structure approach necessarily leads to large matrices.

The class of optimality criteria methods covers a number of different strategies (see e.g., [13,22]). They are based upon stress criteria, displacement criteria or the Kuhn–Tucker necessary conditions of optimality. Although optimality criteria procedures in general have proven to be efficient in topology design there might be problems concerning convergence and stability.

The homogenization method [4] is based on using composite materials to model local material properties. A homogenized strain energy is utilized to formulate a material design problem whose solution can be interpreted as the topology of a discrete structure. The method is limited with respect to the choice of the objective function. This problem can be alleviated using heuristic variations of the homogenization method like the solid isotropic microstructures with penalization method (SIMP) [3]. Here the density distribution of

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the material is modified until a quasi-discrete structure is achieved.

The evolutionary structural optimization (ESO) of Xie and Steven [25,26] is a simple method based on so-called rejection criteria which are used to remove inefficient material in a structure. Despite the notion "evolutionary" the method has no biologically inspired steps based on the Darwinian mutation and selection principle and should not be mixed up with evolution strategies or genetic algorithms, see Section 2.2. As the homogenization method ESO is not capable to be used with arbitrary objective functions.

In order to overcome the above-mentioned flaws stochastic topology optimization has gained interest in the last years [2,9,10,18,19].

Stochastic optimization is a very general concept. The respective methods do not have any demand regarding the formulation of the objective function. The stochastic topology optimization methods are not restricted to mechanical problems but can be used with arbitrary objective functions, see e.g., the applications to the optimization of artificial neural networks or the digital filter design in [14].

In contrast to the classical approaches, the alternative methods described in this paper allow also for insertion of bars (see [21, p. 43–44]). This is of importance, because mechanical structures from engineering practice may have many joints and starting from highly connected structures can be computationally demanding. A completely connected initial structure is only one of many possible starting points in the search space. Almost arbitrary initial structures (for example structural models, which have emerged from a conventional design process) can be used if the structural variations allow for substantial structural rearrangements, that means addition and deletion of bar elements. The relevance of this feature increases with the number of joints if the assembling of the stiffness matrix is supported by appropriate data structures.

2. Stochastic optimization

2.1. Simulated annealing

In a famous paper Metropolis et al. [16] introduced a method, which allows the computational simulation of physical systems in thermal equilibrium. Kirkpatrick et al. [12] have taken up the Metropolis approach and adapted to the solution of complex optimization problems (simulated annealing method, SA). The idea behind simulated annealing is based on the close correspondence of energy in statistical mechanics and cost or system quality in optimization problems. Since physical systems can be forced into the energetic ground state by a careful annealing process, an optimization problem can be driven towards the global optimum by adjusting a parameter, which can be considered as the counterpart to the physical temperature. Determining the proper annealing (cooling) schedule for a given problem can be demanding.

2.2. Evolution strategies

In his pioneering evolution theory Darwin gave as the reason for the development of species the principle *survival of the fittest*. This principle states that only by natural selection an optimal adaptation of a particular species to the environment and living conditions could occur.

It is obvious to use such a selection principle as the basis for optimization methods (evolutionary algorithms). For this purpose the variable vector of the optimization problem can be interpreted as an individual of an artificial population and the selection can take place on the basis of the associated objective function value. The objective function plays the role of the fitness in a simulated environment and the adaptation to these conditions leads to the solution of the underlying optimization problem.

The formulation and algorithmic realization of such evolutionary algorithms goes back to Rechenberg [20] and Schwefel [23,24] who developed the so-called evolution strategies (ES), and Holland [11] who laid the foundation for the genetic algorithms (GA). Since in a previous study [2] GA methods turned out to be not very efficient for the problems at hand they have not been given further attention in this paper. In the following a short description of the evolution strategies will be given.

Two main variants of the evolution strategies are in use. Both assume that in each iteration step a population of μ parental vectors exists. Then, with the aim of generating an offspring vector, a parental vector is chosen randomly and modified by adding a random variation (mutation). This procedure is repeated until λ offsprings have been created. In the $(\mu + \lambda)$-ES an intermediate population of the $\mu + \lambda$ individuals is the basis for the selection of the μ best vectors to be parents of the next iteration. In the so-called (μ, λ)-ES, the μ best vectors will be taken out of the λ offsprings only $(\lambda > \mu)$.

Further biological phenomena like recombination, migration or the competition between populations can be easily included in such an evolutionary optimization concept.

There is an extensive literature on the theoretical background and recommendations concerning the practical application of evolution strategies (see e.g., [1,7,24]). However, these theoretical investigations are mainly devoted to continuous problems. Relatively few results concerning the strategy parameters exist with respect to discrete problems. On the other side it is