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Stochastic homogenized effective properties of three-dimensional composite material with full randomness and correlation in the microstructure

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ABSTRACT

In this work, random homogenization analysis for the effective properties of composite materials with unidirectional fibers is addressed by combining the equivalent inclusion method with the Random Factor Method (RFM). The randomness of the micro-structural morphology and constituent material properties as well as the correlation among these random parameters are fully considered, and stochastic effective properties including effective elastic tensor and effective elastic properties together with their correlation are sought. Results from the RFM and the Monte-Carlo Method (MCM) are compared, and the impact of randomness and correlation of the micro-structural parameters on the random homogenized results are investigated by the two methods. Finally, the correlation coefficients of the effective properties are obtained by the MCM. The RFM is found to deliver rapid results with comparable accuracy to the MCM approach.

1. Introduction

Heterogeneous materials [1] including, e.g., composite and functionally graded materials are increasingly used in different fields [2,3], consisting of the aerospace, automotive and civil construction industries, whereby they are designed and employed to satisfy special functional requirements that conventional homogeneous materials cannot meet. For heterogeneous materials under sustained mechanical and thermal stresses, micro-structural features exert an important and often controlling influence on the overall performance by affecting several mechanical properties [4]. Thus current attempts to increase strength, stiffness, ductility and durability of materials and structures require a full appreciation and characterization of their micro-structural properties [5].

Homogenization techniques are widely used to compute the effective properties of heterogeneous materials based on the knowledge of geometry and material properties of their microstructure. These techniques are both of computational and analytical nature. For analytical techniques, early approximations for this purpose were presented by Voigt [6] and Reuss [7]. Later, key advances were reached with the work of Eshelby [8] and Hashin and Shtrikman [9]. Additional classical models to estimate the effective properties included the self-consistent method, the dilute distribution method, the Mori and Tanaka method [10] and many others, see e.g. Aboudi [11], Mura [12] and Nemat-Nasser and Hori [13]. Computational technique based on multiscale finite element method was developed as well, see e.g. Zohdi and Wriggers [14], Stroeven et al. [15], and Vel and Goupee [16], and Temizer and Wriggers [17].

It is well known that geometry and material parameters can never be determined with absolute certainty [18]. This recently motivated an increasing attention to random heterogeneous materials, including, e.g., composite materials with uncertainty in the location/shape of the reinforcement and/or in the pore/particle spatial distribution in the matrix as well as in the mechanical properties of the components. Many progresses about random homogenization were reached with the work of many scholars. Kamiński and Szafran [19] applied the generalized stochastic perturbation-based finite element method to computational modeling of

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random interface defects in composite materials, where the number of defects as well as their radii and the Young modulus of the matrix were taken as Gaussian random variables. Sakata and coworkers [20] presented a perturbation-based stochastic homogenization analysis method for the thermal expansion coefficient of a fiber-reinforced composite material. Feng and Li [21] proposed a robust and efficient algorithm based on nonlinear transformation of Gaussian random fields to reconstruct two-phase composite materials with random morphology, according to given samples or given statistical characteristics. Milani and Lourenço [22] presented a kinematic rigid-plastic homogenization model for the limit analysis of masonry walls arranged in random texture and out-of-plane loaded, where blocks constituting a masonry wall are supposed infinitely resistant with a Gaussian distribution of height and length. Xu et al. [23] formulated a Green-function-based multiscale method to decompose a boundary value problem with random microstructure into a slow scale deterministic problem and a fast scale stochastic one by employing generalized variational principles.

Despite the progress summarized above, however, analytical homogenization or computational homogenization of heterogeneous materials with uncertainty in microstructure still remains an enormous challenge. The existing models mainly address the randomness of the micro-structural morphology [14,16,17,21–23] or sometimes of several material properties [19,20,24]. Moreover, they do not include the correlation of the micro-structural properties or the morphology parameters as well as the correlation among the random homogenized results. Compared with the randomness of micro-structural parameters and homogenized results, the correlation existing in random micro-structural parameters or homogenized results is very important as well since sometimes an unknown parameter or property can be estimated to a certain extent starting from the known parameter or property according to their correlation.

The goal of this work lies in tackling the stochastic homogenization problem of composite material by a convenient approach when fully considering the uncertainty including the randomness and correlation in microstructure. Herein, the Random Factor Method (RFM) proposed in [25,26] is extended to the computation of the random homogenized effective properties of a unidirectional fiber reinforced polymer (FRP) composite with orthotropic behavior at the macroscale which is a classic model in composite material. The stochastic macroscopic effective properties consisting of effective elastic tensor $\mathbf{E}^H$ and effective elastic properties (Young's elastic moduli E_x^H, E_y^H, E_z^H, Shear elastic moduli G_{yz}^H, G_{zx}^H, G_{xy}^H as well as Poisson ratios ν_{yz}^H, ν_{zx}^H, ν_{xy}^H derived from $\mathbf{E}^H$) of FRP composites are formulated by combining the analytical homogenization approach with the RFM, whereby the randomness of the material properties and morphology parameters of the two constituents as well as the correlation among these random variables are simultaneously taken into account. Moreover, all results obtained from the RFM are compared with those from Monte-Carlo Method (MCM) in order to confirm the validity of the proposed method. Finally, the correlation in the macroscopic effective properties is acquired by the MCM.

2. Stochastic homogenization of macroscopically orthotropic composite media

2.1. Homogenized effective properties for orthotropic materials

The equivalent inclusion method is one of the effective methods for estimating the homogenized elastic tensor of composite materials. For a unidirectional fiber reinforced composite material, a formula from the Mori–Tanaka theory [10] based on the equivalent inclusion idea can be used for the estimation. According to this formula, the homogenized effective elastic tensor $\mathbf{E}^H$ is computed as:

$$\mathbf{E}^H = \mathbf{E}_m\mathbf{X}^{-1}\mathbf{Y} = \begin{bmatrix} E_{11}^H & E_{12}^H & E_{13}^H \\ E_{21}^H & E_{22}^H & E_{23}^H \\ E_{31}^H & E_{32}^H & E_{33}^H \end{bmatrix} \tag{1}$$

$$\mathbf{X} = \mathbf{E}_m - (1 - V_f)(\mathbf{E}_m - \mathbf{E}_f)\mathbf{S} \tag{2}$$

$$\mathbf{Y} = \mathbf{E}_m - (\mathbf{E}_m - \mathbf{E}_f)\{\mathbf{S} - V_f(\mathbf{S} - \mathbf{I})\} \tag{3}$$

where $\mathbf{E}_m$ and $\mathbf{E}_f$ are respectively the elastic tensor of the matrix material and fiber inclusions, V_f is the volume fraction of the fibers, $\mathbf{S}$ is the Eshelby matrix and $\mathbf{I}$ is the 6×6 unit matrix. Assuming both constituents (matrix and fiber) are isotropic materials, their elastic matrix can be expressed as

$$\mathbf{E} = \frac{e(1-\nu)}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1 & \frac{\nu}{1-\nu} & \frac{\nu}{1-\nu} & 0 & 0 & 0 \\ \frac{\nu}{1-\nu} & 1 & \frac{\nu}{1-\nu} & 0 & 0 & 0 \\ \frac{\nu}{1-\nu} & \frac{\nu}{1-\nu} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2(1-\nu)} \end{bmatrix} \tag{4}$$

where e and $\boldsymbol{\nu}$ are respectively the Young's modulus and the Poisson ratio of the material. In the following, these properties will be denoted as e_m and $\boldsymbol{\nu}_m$ for the matrix, and e_f and $\boldsymbol{\nu}_f$ for the fibers. The Eshelby matrix $\mathbf{S}$ is a 6×6 matrix and depends on the shape of the inclusions. In case of long continuous unidirectional fibers the non-zero elements of $\mathbf{S}$ are respectively: $S_{1111} = \frac{1}{2(1-\nu_m)}\left[\frac{a_2^2+2a_1a_2}{(a_1+a_2)^2} + (1-2\nu_m)\frac{a_2}{a_1+a_2}\right]$, $S_{1133} = \frac{1}{2(1-\nu_m)}\frac{2\nu_m a_2}{a_1+a_2}$, $S_{1122} = \frac{1}{2(1-\nu_m)}\left[\frac{a_2^2}{(a_1+a_2)^2} - (1-2\nu_m)\frac{a_2}{a_1+a_2}\right]$, $S_{2323} = \frac{a_1}{2(a_1+a_2)}$, $S_{2211} = \frac{1}{2(1-\nu_m)}\left[\frac{a_1^2}{(a_1+a_2)^2} - (1-2\nu_m)\frac{a_1}{a_1+a_2}\right]$, $S_{2222} = \frac{1}{2(1-\nu_m)}\left[\frac{a_1^2+2a_1a_2}{(a_1+a_2)^2} + (1-2\nu_f)\frac{a_1}{a_1+a_2}\right]$, $S_{2233} = \frac{1}{2(1-\nu_m)}\frac{2\nu_m a_1}{a_1+a_2}$, $S_{3131} = \frac{a_2}{2(a_1+a_2)}$, $S_{1212} = \frac{1}{2(1-\nu_m)}\left[\frac{a_1^2+a_2^2}{2(a_1+a_2)^2} + \frac{1-2\nu_m}{2}\right]$, where a_1 and a_2 are the cross-sectional fiber dimensions, see Fig. 1 [24], where $a_3 \gg a_1, a_2$.

Several types of industrial materials can be regarded as an isotropic or orthotropic material, and the common material properties such as Young's modulus or Poisson ratio for each direction will be used for evaluation of material characteristics. The homogenized effective elastic properties of isotropic or orthotropic composite materials can be computed by the homogenized compliance. For orthotropic materials, e.g. the composite shown in Figs. 1 and 2 [24], the homogenized elastic properties can be computed as follows:

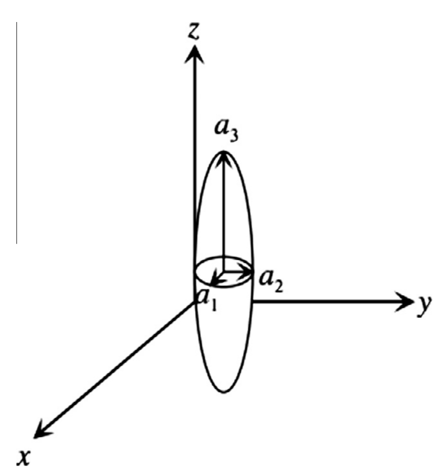

Fig. 1. An fiber inclusion.