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# Local force identification on flexural plates using reduced Finite Element models

Cédric Renzi [a,*], Charles Pézerat [b], Jean-Louis Guyader [a]

[a] Laboratoire Vibrations Acoustique, INSA-LYON, 25 bis avenue Jean Capelle, bat. St Exupéry, F-69621 Villeurbanne Cedex, France
[b] Laboratoire d'Acoustique de l'Université du Maine, CNRS, avenue Olivier Messiaen, F-72085 LE MANS Cedex 09, France

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ABSTRACT

The identification of excitations on plates by an inverse method using local Finite Element modeling is studied. A dynamic condensation is proposed in order to eliminate Degrees Of Freedom not directly measurable like rotations. Before the condensation, the Craig–Bampton method can be optionally used to reduce the model (decrease of both computational and measurement times). Regularization based on the Tikhonov method and a double inversion of the operator is performed. After a numerical study of the accuracy of each step for different excitations, an experimental validation finally proved that it was both possible to locate accurately a shaker over a wide range of frequency and to estimate the amplitude of the injected force. Both dynamic condensation and Craig–Bampton reduction were used. Even if it induces some imprecision, the Craig–Bampton reduction was found to be a fundamental step to reduce both computational cost and the measurement effort which were the main problems to address in such an approach.



## 1. Introduction

Many vibroacoustic studies use Finite Element modeling in order to predict the dynamic behavior of structures. Current software and computing resources allow engineers to create numerical models that can be applied for more complex structures and for higher frequencies. In practice, the end-user must define the geometry of the structure, the material characteristics and the boundary conditions and excitations. The first two types of data are usually provided by the designers and are refined by experimental updating, whereas boundary conditions and excitations are less well-defined. Prediction accuracy is then affected by poor knowledge of the latter two types of data. In terms of experimental updating, the purpose of this paper is to define an experimental approach to identify excitations on the basis of a model and accessible measurements. The aim is to reverse the usual application of the model by calculating sources from the measurements of their effects. This is exactly the same philosophy as that used in the Force Analysis Technique (FAT) [1], also known as the RIFF method (from the French definition "Résolution Inverse Filtrée Fenêtrée"), developed several years ago for simple structures like beams [1–3], plates [3] and shells [4,5]. In the Force Analysis Technique, the calculation of the exciting force distribution is obtained by using the analytic equation of motion, where spatial derivatives are approximated by finite differences. The principal interest of this method is the fact that the equation of motion is verified locally only and does not have to be solved. In other words, knowledge of the equation of motion for which measurements are performed is sufficient. Boundary conditions and effects due to excitations outside the measurement area can be ignored. The FAT is also highly sensitive to errors in measurement data, so that a regularization process has been developed through low-pass wavenumber filtering. In [6], the authors presented an equivalent regularization, using a double inversion of the operator of the structure, where the second inversion is regularized by the classic Singular Value Decomposition. The principal limitation of the FAT is the use of an analytic equation of motion. Applications are restricted to test benches [2,7] or to cowls [3] which can be modeled with the Love–Kirchhoff Theory [8].

The aim of this study is to adapt the FAT for application to more industrial cases where the use of Finite Element Modeling is unavoidable. Other authors have used Finite Element (FE) models to perform force identification, but with different aims. Busby developed an approach using a Tikhonov regularization and a static condensation of rotations on beams in the time domain, with assumptions on the force locations [9]. Ibrahim and Sestieri used a Finite Element model of a set of beams using dynamic condensation,

* Corresponding author.
*E-mail addresses:* cedric.renzi@2003.ecam.fr (C. Renzi), charles.pezerat@univ-lemans.fr (C. Pézerat).

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**Nomenclature**

*Operators, vectors and matrices related to the original Finite Element (FE) model*

- **M** FE Mass matrix of the structure
- **K** FE Stiffness matrix of the structure
- **L** FE operator depicting the dynamics of the free structure
- **V** FE vector of the nodal vibratory data (translations and rotations)
- $\mathcal{A}$ FE vector of the exterior nodal mechanical actions (forces and moments)
- $\mathcal{F}$ FE vector of the exterior nodal forces
- $\mathcal{M}$ FE vector of the exterior nodal moments

*Operators and vectors after Craig–Bampton reduction*

- $\mathbf{L^r}$ reduced FE operator
- $\mathbf{V^r}$ reduced FE vector of the nodal vibratory data
- $\mathbf{A^r}$ reduced FE vector of the exterior nodal mechanical actions

*Operators and vectors after dynamical condensation of non measured Degrees Of Freedom*

- $\mathbf{L^c}$ condensed FE operator
- $\mathbf{V^c}$ condensed FE vector of the nodal vibratory data
- $\mathbf{F^c}$ condensed FE vector of the exterior nodal mechanical actions

*Operators and vectors after Craig–Bampton reduction and dynamical condensation of non measured Degrees Of Freedom*

- $\mathbf{L^{rc}}$ reduced then condensed FE operator
- $\mathbf{V^{rc}}$ reduced then condensed FE vector of the nodal vibratory data
- $\mathbf{F^{rc}}$ reduced then condensed FE vector of the exterior nodal mechanical actions

*Operators and vectors after regularization*

- $\mathbf{L^{ctikh}}$ regularized pseudo-inverse of the inverse of $\mathbf{L^c}$
- $\mathbf{F^{ctikh}}$ vector of the identified forces with dynamical condensation of the non measured Degrees Of Freedom (regularised solution)
- $\mathbf{L^{rctikh}}$ regularized pseudo-inverse of the inverse of $\mathbf{L^{rc}}$
- $\mathbf{F^{rctikh}}$ vector of the identified forces with Craig–Bampton reduction and dynamical condensation of the non measured Degrees Of Freedom (regularized solution)

by keeping the locations of a priori forces as master Degrees Of Freedom (DOF) [10]. Corus and Balmès used a Finite Element approach to tune their model with force prelocalization and a projection of the model in the truncated modal basis of the complete structure [11,12].

The approach presented in the present paper is aimed at quantifying and localizing the forces from an FE model with as few assumption as possible. The first approach was developed on beams [13,14], showing the possibility of using a Finite Element approach locally (on only part of the structure) with free boundary conditions. The exciting forces inside the area studied are well-identified as are the coupling forces with the rest of the structure at the boundaries. One of the differences with the FAT is that it identifies the coupling forces at the limits through the use of free boundary conditions in the inverse problem. It is necessary to reduce the model in order to use such a method for 2D or more complex structures. Indeed, experiments and calculations can last too long when the number of DOF starts getting very high, making it necessary to study the effects of reductions.

In this paper, after the presentation of the general principle of this inverse problem, a dynamic condensation is proposed in order to eliminate rotations which are considered as non accessible DOF. To further reduce the number of DOF and accelerate computing time, we then propose performing a Craig–Bampton reduction prior to the dynamic condensation. This add-on is presented in this paper as an option. As in [13], the regularization method chosen is performed by a double inversion in which the Tikhonov regularization is applied to the second inversion. After describing all these steps in the first section, the numerical simulations are presented in the second section. The advantage of the simulations is that they permit testing the principle of the inverse method using exact data, and testing the regularization using noisy data. Finally, the third section is devoted to the experimental validation of the method. The setup corresponds to an L-Shaped Plate with arbitrary boundary conditions and excited by a shaker. The location is clearly identified with and without the Craig–Bampton option and the force spectrum identified with the Craig–Bampton option appears comparable with the direct measurement of the exciting force.

## 2. Methodology development

### 2.1. Illustration of the principles of the approach

The objective of this subsection is to provide an illustration of the method and of the effects of its idealizing assumptions on the obtained results. A flexural plate of thickness $h$ is considered to be in harmonic motion at an angular frequency $\omega$. Its material properties are $\rho$, mass density, $\eta$, damping factor for linear hysteretic damping model and $E$, the Young's modulus modeling local stiffness. In the following, the plate is in the $\left(\vec{x}, \vec{y}\right)$ plane. In order to model the local behavior, 3D-geometry shell elements with a Kirchhoff formulation [15–17] are used. Normals to the shell elements are defined along the $+\vec{z}$ axis. DOF of the $i$th node are limited to:

- the transverse translation $w_i$ along $+\vec{z}$ (associated with the force $\mathcal{F}_\mathbf{i}$),
- the rotation $\theta_i^x$ around $+\vec{x}$ (associated with a moment $\mathcal{M}_\mathbf{i}^\mathbf{x}$),
- the rotation $\theta_i^y$ around $+\vec{y}$ (associated with a moment $\mathcal{M}_\mathbf{i}^\mathbf{y}$).

The matrix system describing the FE model of a given structure can be written as [15]:

$$\left(-\omega^2\mathbf{M} + (1 + j\eta)\mathbf{K}\right)\mathbf{V} = \mathbf{LV} = \mathcal{A}, \tag{1}$$

where $j$ is the imaginary number, **M** symbolizes the mass matrix of the structure, **K** is the dynamic stiffness matrix of the structure, **L** is the Finite Element operator of the structure, **V** is the vector of responses containing $N$ nodal displacements and $2N$ nodal rotations and $\mathcal{A}$ is the vector of mechanical actions containing $N$ nodal forces $\mathcal{F}$ and $2N$ nodal moments $\mathcal{M}$.

The mass and stiffness matrices of the structure are computed using the assembly of elementary matrices evaluated by the interpolation of the displacement field over the elementary domain using quadratic shape functions [15].

For plates and shells, standard elementary topologies are based on quadrangles (quad4 elements) and triangles (tria3 elements). In