Computers and Structures 141 (2014) 9–18

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

Hybrid uncertain analysis for the prediction of exterior acoustic field with interval and random parameters

Ning Chen, Dejie Yu *, Baizhan Xia

State Key Laboratory of Advanced Design and Manufacturing for Vehicle Body, Hunan University, Changsha, Hunan 410082, People's Republic of China

ARTICLE INFO

Article history:
Received 2 January 2014
Accepted 12 May 2014
Available online 11 June 2014

Keywords:
Exterior acoustic field prediction
Random interval response
Perturbation method
Random moment method

ABSTRACT

This paper presents a hybrid perturbation method (HPM) for the prediction of exterior acoustic field with interval and random variables. By regarding the interval variables as constants, the random interval matrix and vector are firstly expanded by the first-order Taylor series. Then the expectation and variance of response vector can be calculated based on the matrix perturbation theory and random moment method. At last, the bounds of expectation and variance of the response vector can be obtained by the interval perturbation method. Two numerical examples are given to illustrate the feasibility and effectiveness of the proposed method.

1. Introduction

In recent years, the performance of NVH (Noise, Vibration and Harshness) has received considerable attention. Researches on the acoustic behavior of structural–acoustic systems have been undergone a rapid development in engineering. At present, the finite element method (FEM), the boundary element method (BEM) and the infinite element (IEM) are the most popular and commonly used numerical approaches for the low-frequency acoustic radiation problems. Traditional acoustic radiation problems have been analyzed under the assumption that the physical properties, the applied loads and the boundary conditions are deterministic. However, due to the effects of manufacturing or construction tolerances, aggressive environment factors and unpredictable external excitations, uncertainties associated with material properties, geometric dimensions, applied loads and other parameters are unavoidable. So far, the acoustic radiation problem with uncertain parameters has not been researched yet. The probabilistic approaches are usually employed to handle the uncertainty. In probabilistic approaches, the uncertain parameters are treated as random variables and the probability density functions are defined unambiguously [1–5]. Up to now, The Monte Carlo method (MCM) [6–9], perturbation based stochastic finite element method [10–13], spectral stochastic finite element method [14,15] and other types of stochastic methods [16] have been developed to deal with random problems. Recently, the Bayesian approach has been introduced to treat uncertain problems [17,18].

Probabilistic methods are only applicable when the probability density function of an uncertain parameter is available. Unfortunately, in many practical applications, it is hard or costly to construct the precise probability density function as the objective information of uncertain parameters is limited. Thus, the non-probabilistic approaches are the suitable alternatives. Ben-Haim and Elishakoff [19] proposed the non-probability convex model to deal with the uncertain problems without sufficient information. For the non-probability convex model, the least and most favorable responses of uncertain problems can be obtained by employing an anti-optimization approach [20–22]. Interval analysis, the other alternative non-probabilistic approach, has achieved significant progress over the past decade. A lot of methods [23–29] have been proposed to reduce the overestimation, which caused by the dependency phenomenon in interval analysis.

When one comes to the hybrid uncertain engineering problems with random and non-probabilistic parameters simultaneously, a desirable hybrid analysis framework can be constructed. The stochastic interval responses of structures with uncertain-but-bounded parameters under random excitation have been investigated in literatures [30,31]. The hybrid perturbation Monte–Carlo method (HPMCM) has been studied by Gao et al. for the response analysis of hybrid uncertain structures with random and interval parameters [32]. The reliability analysis of hybrid uncertain structures with random and non-probabilistic parameters has attracted more and more attention [33–36]. Recently, to avoid the excessive computational cost of HPMCM suffering from Monte–Carlo method, the hybrid perturbation vertex method for the hybrid uncertain structure-acoustic problem with random and interval parameters has been developed by Xia et al. [37].

* Corresponding author. Tel.: +86 731 88821915; fax: +86 731 88823946.
E-mail address: djyu@hnu.edu.cn (D. Yu).

http://dx.doi.org/10.1016/j.compstruc.2014.05.004
0045-7949/

As mentioned above, research on the hybrid uncertain models is still in its preliminary stage and has not been applied to the low-frequency acoustic radiation problems. The parameters of numerical models are treated as the deterministic values in traditional acoustic radiation analysis [38,39]. This paper proposed a hybrid perturbation method (HPM) for the response prediction of hybrid uncertain exterior acoustic fields with random and interval variables. The random variables are employed to deal with the uncertain parameters with sufficient information to construct the probability distributions, whereas, the interval variables are employed to deal with the uncertain parameters without sufficient information to construct the probability distributions. According to the random and interval analysis, the random interval dynamic equilibrium equation of the hybrid uncertain exterior acoustic field is established. By regarding the interval variables as constants, the random interval matrix and vector are firstly expanded by the first-order Taylor series at the expectations of random variables. Then, the expectation and variance of response vector can be calculated based on the matrix perturbation theory and the random moment method. At last, the bounds of expectation and variance of the response vector can be obtained by the interval perturbation method.

The remainder of this paper is organized as follows. In Section 2, the equilibrium equation for exterior acoustic field prediction is established. In Section 3, a hybrid perturbation method to calculate the bounds of expectation and variance of the response vector of exterior acoustic field with random and interval parameters is proposed. Two numerical examples are provided in Section 4 and some conclusions are given in Section 5.

2. FEM/BEM equilibrium equation of exterior acoustic field

In this paper, the coupled FEM/BEM method is considered to handle the exterior acoustic field prediction, in which the FEM and BEM models are used to simulate the structure and the fluid, respectively. Both the structure and the acoustic medium satisfy the linear constitutive equations and the acoustic medium is assumed to be inviscid and incompressible. On the interface between the structure and the fluid, only the normal displacement of the structure is coupled with the fluid and the fluid just exerts normal loads on the structure.

2.1. FEM model of the shell structure

In the frequency domain, the steady-state finite element equation of the shell structure can be expressed as

$$(\boldsymbol{K}_s + i\omega \boldsymbol{C}_s - \omega^2 \boldsymbol{M}_s)\boldsymbol{u_s} = \boldsymbol{F}_s \tag{1}$$

where $\boldsymbol{K}_s$ is the structural stiffness matrix, $\boldsymbol{C}_s$ is the structural damping matrix, $\boldsymbol{M}_s$ is the structural mass matrix, $\boldsymbol{F}_s$ is the exciting force vector, ω is the angular frequency of exciting force, $i = \sqrt{-1}$ is an imaginary unit, $\boldsymbol{u}_s$ is the displacement vector.

The Rayleigh damping model for the attached damping layer is employed, $\boldsymbol{C}_s$ can be written as

$$\boldsymbol{C}_s = \alpha \boldsymbol{M}_s + \beta \boldsymbol{K}_s \tag{2}$$

where α and β are damping coefficients of the damping material.

The structural stiffness matrix $\boldsymbol{K}_s$ can be expressed as

$$\boldsymbol{K}_s = \int_{\Omega_s} \boldsymbol{B}_m^{\mathrm{T}} \boldsymbol{D}_m \boldsymbol{B}_m \mathrm{d}\Omega + \int_{\Omega_s} \boldsymbol{B}_b^{\mathrm{T}} \boldsymbol{D}_b \boldsymbol{B}_b \mathrm{d}\Omega + \int_{\Omega_s} \boldsymbol{B}_s^{\mathrm{T}} \boldsymbol{D}_s \boldsymbol{B}_s \mathrm{d}\Omega \tag{3}$$

where $\boldsymbol{B}_m, \boldsymbol{B}_b$ and $\boldsymbol{B}_s$ are the membrane stress shape function, the bending stress shape function and the shear stress shape function of the shell element; $\boldsymbol{D}_m$, $\boldsymbol{D}_b$ and $\boldsymbol{D}_s$ are the membrane stiffness constitutive coefficient matrix, the transverse shear stiffness constitutive coefficient matrix and the bending stiffness constitutive coefficient matrix; Ω_s is the structural domain.

The structural mass matrix $\boldsymbol{M}_s$ can be expressed as

$$\boldsymbol{M}_s = \int_{\Omega_s} \boldsymbol{N}_s^{\mathrm{T}} \boldsymbol{m} \boldsymbol{N}_s \mathrm{d}\Omega \tag{4}$$

where $\boldsymbol{m}$ is the element mass matrix, $\boldsymbol{N}_s$ is the Lagrange shape function of the isoparametric quadrilateral element.

The load vectors $\boldsymbol{F}_s$ can be expressed as

$$\boldsymbol{F}_s = \int_{\partial\Omega_{sf}} \boldsymbol{N}_s^{\mathrm{T}} \tau \mathrm{d}\Gamma + \int_{\Omega_s} \boldsymbol{N}_s^{\mathrm{T}} b_s \mathrm{d}\Omega \tag{5}$$

where τ is the surface traction, b_s is the body load, Ω_{sf} is the coupled interface.

In the stead-state form, the structural velocity vector $\boldsymbol{v}$ and displacement vector $\boldsymbol{u}_s$ satisfy the following relationship

$$\boldsymbol{v} = i\omega \boldsymbol{u_s} \tag{6}$$

Multiplying both sides of Eq. (1) with $i\omega$, the Fourier transformation of the dynamic equilibrium equation about the structural vibration velocity can be obtained as

$$(\boldsymbol{K}_s + i\omega \boldsymbol{C}_s - \omega^2 \boldsymbol{M}_s)\boldsymbol{v_s} = i\omega \boldsymbol{F}_s \tag{7}$$

2.2. The formulation of acoustic BEM

In the frequency domain, the acoustic steady-state pressure p is governed by the Helmholtz equation

$$\nabla^2 p + k^2 p = 0 \tag{8}$$

where $k = \omega/c$ represents the wavenumber, ω is the angular frequency, c is the sound speed, and $\nabla = \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k$.

The classical boundary Helmholtz integral equation can be expressed as following

$$C(P)p(P) = \int_S \left(i\rho_f \omega v_n G_0 + p\frac{\partial G_0}{\partial \boldsymbol{n}} \right) ds \tag{9}$$

Here, the symbol P denotes the objective field point where the sound pressure will be calculated and C is the interpolation coefficient. The integration of Eq. (9) is performed along the structural surface S, where the symbol ρ_f, ω and v_n represent the mass density, frequency and normal velocity of the acoustic medium, respectively. The symbol G_0 denotes the Green function which is the fundamental solution of the Helmholtz equation under the unit impulse. The symbol $\boldsymbol{n}$ is the normal vector of the structural surface and points to the acoustic domain. The schematic diagram of the classical boundary Helmholtz integral equation is illustrated in Fig. 1.

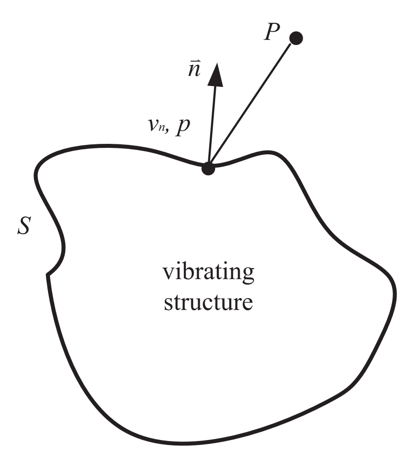

Fig. 1. Schematic illustrating of the boundary integral equation.