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Numerical modeling of elastic waveguides coupled to infinite fluid media using exact boundary conditions

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ABSTRACT

The simulation of guided waves in plate structures and cylinders coupled to infinite fluids is addressed. The approach is based on the Scaled Boundary Finite Element Method. Only a straight line is discretized that represents the through-thickness direction or the radial direction. The surrounding fluid is accounted for by employing a damping boundary condition that is based on the analytical description of the radiation impedance. Since the radiation impedance is a function of the wavenumber in the waveguide, an iterative solution procedure is applied. The algorithm is highly efficient while the results are in agreement with the Global Matrix Method.

1. Introduction

Elastic guided waves can be excited in thin-walled structures, i.e. when the thickness of the solid is in the order of the wavelength of longitudinal and shear waves in the material under consideration. In the ultrasonic range, guided waves offer a large variety of applications in non-destructive testing [1–3], structural health monitoring [4–6] or material characterization [7–10]. On a very different scale, similar phenomena occur e.g. in soil layers [11,12] or in reservoirs adjoining dams [13] and are considered in geophysics and earthquake engineering [14]. Due to their complex propagation behavior, numerical methods are often employed to model guided waves in a given structure. A crucial step in the analysis of guided waves is the computation of dispersion curves, i.e. the frequency-dependent phase and group velocities of propagating modes. To fulfill this task, numerous approaches that are optimized for particular structures have been developed over the last decades. For homogeneous plates and cylinders, analytical solutions as derived by Lamb [15] and by Pochhammer [16] and Chree [17], respectively, can be obtained. If the plate or pipe consists of several layers, the Global Matrix Method (GMM) [18] that is based on the analytical description of the reflection and transmission at each interface, is often employed. The GMM can easily be applied to simple structures consisting of few layers. Then again, the solution becomes very cumbersome if many layers are present or if material damping is considered.

Contrary to the analytical models, different numerical approaches have been applied, most of them being based on the Finite Element Method. A full three-dimensional Finite Element model is capable of describing waveguides of arbitrary shape and material properties, but leads to very high computational costs [19]. In geophysics, the Thin Layer Method (TLM) is widely used [20,21]. It is based on a discretization of the through-thickness direction of a two-dimensional structure, while the direction of wave propagation is described analytically. The Semi-Analytical Finite Element (SAFE) Method [22–25] uses the same concept for the simulation of guided waves in the ultrasonic range. It has been extended to three-dimensional waveguides by discretizing the two-dimensional cross-section with traditional Finite Elements.

Recently, a particular formulation of the Scaled Boundary Finite Element Method (SBFEM) [26,27] has been derived for the simulation of guided waves. The SBFEM is a very general semi-analytical method that can be utilized in a wide range of applications to model bounded or unbounded domains in the frequency as well as in the time domain [28–31]. Only the boundary of the computational domain is discretized in the finite element sense, while an analytical formulation is used to scale the mesh in the interior of the domain or towards infinity, respectively. This method has been used to model guided wave propagation in the time domain very efficiently [31]. Applying the concept of the SBFEM to the computation of dispersion curves [32–35] leads to a formulation that shows similarities with the TLM and the SAFE methods, while

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different solution procedures are applied in order to enhance efficiency [36]. Moreover, computational costs have been drastically reduced by employing spectral elements of very high order [33,36].

For a long time it has been considered a major drawback of the mentioned numerical approaches that the modeling of waveguides in contact with an infinite solid or fluid medium is not straightforward. The most common attempts to address this problem are based on absorbing regions [37–40], infinite elements [41,42], perfectly matched layers [43,44] or non-reflecting boundaries [45]. However, these techniques can raise high computational costs, since they require the discretization of a significant part of the surrounding medium. Moreover, the desired solutions of guided wave modes have to be separated from unphysical modes in the surrounding medium, which can be cumbersome. In a very recent development, the Finite Element mesh of the waveguide is coupled to a Boundary Element approach to describe a surrounding fluid [46], leading to a nonlinear eigenvalue problem for the wavenumbers. The Global Matrix Method is theoretically capable of describing infinite media [18]. However, the solution of the characteristic equation is difficult, since complex-valued roots have to be computed due to the attenuation effect of the surrounding medium. Recently, an improved mode-tracing and root-finding algorithm has been proposed [47]. It enhances the reliability by utilizing the interval Newton method and algorithmic differentiation but again leads to high computational costs.

In the SBFEM, the simulation of waveguides that are embedded in an infinite solid medium has recently been addressed by employing a simple dashpot boundary condition on the waveguide's surface [48]. The boundary condition replaces the surrounding medium by a damper and thus accounts for the leakage of waves into the surrounding medium. Though the formulation is approximate, it has been demonstrated that this approach yields very accurate results for practical applications. The computational costs are reduced by several orders of magnitude compared with the application of absorbing regions, since the surrounding medium is not discretized. For the same reason, the solution is straightforward and the identification of propagating modes is trivial.

If the surrounding medium is a fluid, the simple dashpot approach cannot be applied in the same way. The reason is that the effect of the surrounding fluid strongly depends on the direction of waves propagating into the fluid [49,50]. In the current paper, an improved boundary condition that is based on the exact radiation impedance [51–54] is employed. The formulation is valid for perfect fluids, where viscosity is neglected and hence no shear stresses occur in the surrounding medium. In case of plate structures, the boundary condition is similar to the simple dashpot boundary with a correction that accounts for the direction of propagation. For cylinders, an additional factor that consists of the Hankel functions and accounts for the surface curvature, has to be included.

While the derivation of the boundary conditions and their integration into the Scaled Boundary Finite Element equation is straightforward, the solution of the resulting matrix equations is not trivial. The boundary condition is a function of the unknown wavenumber in the waveguide, thus an iterative solution procedure is required. We propose a solution technique based on inverse iteration [55], that has previously been applied in a different context to compute a subset of modes in a waveguide in vacuum very efficiently [36]. At each step of the iteration, the boundary condition is updated and the corresponding eigenvalue is improved until a converged solution is obtained.

Results are presented for a plate and cylinder and the solutions are compared with the commercial software *disperse* [56] that is based on the Global Matrix Method.

2. Fundamental equations for waveguides in vacuum

The scaled boundary finite element formulation for guided waves in elastic waveguides with stress-free surfaces has been detailed in previous publications [31–36,57,58]. Only a very brief summary of the required equations is presented here. Assuming linear elastodynamics, the stresses $\boldsymbol{\sigma}$ and displacements $\mathbf{u}$ in the waveguide obey the governing equation [59]

$$\mathbf{L}^{\mathrm{T}}\boldsymbol{\sigma} + \omega^2 \rho \mathbf{u} = 0 \tag{1}$$

with the differential operator

$$\mathbf{L} = \begin{bmatrix} \partial_z & 0 & 0 & \partial_x & \partial_y & 0 \\ 0 & \partial_x & 0 & \partial_z & 0 & \partial_y \\ 0 & 0 & \partial_y & 0 & \partial_z & \partial_x \end{bmatrix}^{\mathrm{T}} \tag{2}$$

The frequency and mass density are denoted as ω and ρ, respectively. Stresses and strains are related through Hooke's law with the elasticity matrix $\mathbf{D}$,

$$\boldsymbol{\sigma} = [\sigma_z\ \sigma_x\ \sigma_y\ \tau_{xz}\ \tau_{yz}\ \tau_{xy}]^{\mathrm{T}} = \mathbf{D}\boldsymbol{\varepsilon} \tag{3}$$

The strains follow from the displacements as

$$\boldsymbol{\varepsilon} = [\varepsilon_z\ \varepsilon_x\ \varepsilon_y\ \gamma_{xz}\ \gamma_{yz}\ \gamma_{xy}]^{\mathrm{T}} = \mathbf{L}\mathbf{u} \tag{4}$$

The current work focuses on plates and cylinders, where an exact boundary condition can be derived analytically in order to model the influence of the surrounding fluid. The geometries are presented in Fig. 1. The plate is defined in a Cartesian coordinate system (z, x, y), while the cylinder is formulated in cylindrical coordinates (z, θ, r). Γ denotes the plate surfaces and the side faces of the cylinder, respectively. r_i and r_o are the inner and outer radius of the cylinder. The guided waves are assumed to propagate in the z-direction. The plate is of infinite length in x-as well as z-direction. In case of cylinders, a complex Fourier series is employed in the circumferential direction. Hence, for both plates and cylinders, only

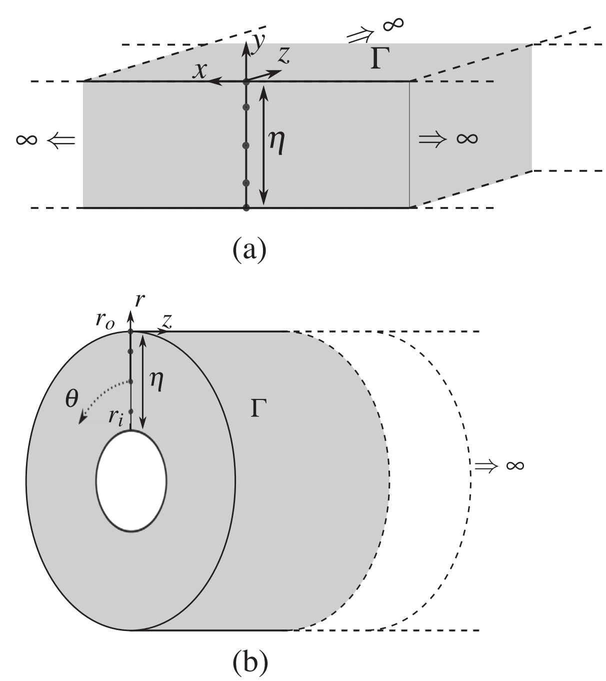

Fig. 1. Discretization of a plate (a) and cylinder (b) using one element of higher order. The element is defined in its local coordinate η. Both structures are assumed to be of infinite dimension in the z-direction. Additionally, the plate is infinite in the x-direction. Γ denotes the plate surfaces and the side faces of the cylinder, respectively. r_i and r_o are the inner and outer radius of the cylinder.