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Generalized shear deformation theory for functionally graded isotropic and sandwich plates based on isogeometric approach

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A B S T R A C T

A generalized shear deformation theory for static, dynamic and buckling analysis of functionally graded material (FGM) made of isotropic and sandwich plates is presented in this paper. Two new distribution functions are proposed in the present formulation. These functions determine the distribution of the transverse shear strains and stresses across the thickness of the plates. The present theory is derived from the classical plate theory (CPT), and hence the shear locking phenomenon can be ignored. It has same number of degrees of freedom as the first order shear deformation theory (FSDT), but it does not require shear correction factors because the shear stress free surface conditions are naturally satisfied. As demonstrated in the following sections, the proposed theory yields very accurate prediction for displacement, stresses, natural frequencies and critical buckling load compared to three-dimensional (3D) elasticity solution. Galerkin weak form of static, free vibration and buckling models for FGM isotropic and sandwich plates are used to create the discrete system of equations. This weak form requires C^1-continuity for generalized displacements. It can be solved by a number of methods such as analytical methods, finite element methods based on the Hermite interpolation functions, meshfree method and recently developed NURBS based isogeometric analysis (IGA). The NURBS basis functions used in IGA are C^{p-1} continuous and therefore can easily satisfy the C^1-continuity condition. Numerical examples are presented to illustrate the effectiveness of the proposed method compared to other methods reported in the literature.

1. Introduction

In the past few decades, development in science and technology has motivated researchers to find new structural materials such as composite and functionally graded materials. These materials have been used in various industrial sectors including aerospace engineering, automotive engineering, civil engineering, nuclear plants and semiconductor technologies. Plates are an important part of many structures. Composite plates are often made of several orthotropic layers bonded together to achieve superior properties such as high stiffness and strength-to-weight ratios, long fatigue life, wear resistance, light weight and a number of other attractive properties. FGM plates belong to a special class of composite plates. In a typical FGM plate, the material properties are tailored by mixing two distinct material phases for example ceramic and metal which are well capable of reducing thermal stresses, resisting high temperature environment and preventing corrosion. In the case of FGM sandwich plates, two types of FGM sandwich plates are commonly used: (a) the sandwich plate with FGM core and two isotropic skins; (b) the sandwich plate with isotropic core and two FGM skins. To use them effectively, a good understanding of bending behavior, stress distribution, dynamic and buckling responses of these plates is necessary.

Besides the extensive application of FGM plates in engineering structures, a large number of plate theories have been developed to analyze the thermo-mechanical behavior of such structures. The classical plate theory (CPT) relied mainly on the Kirchhoff–Love assumptions in order to provide acceptable results for thin plates. However, it may not produce accurate results for moderately thick plates. In order to improve the CPT, first order shear deformation theory (FSDT), the Reissner–Mindlin theory [1–5], which takes shear effect into account, was therefore developed. In addition, with the linear in-plane displacement assumption across the plate thickness, shear strain/stress distribution obtained from FSDT is inaccurate and does not satisfy the traction free boundary conditions at the plate surfaces. The shear

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correction factors (SCF) are hence required to rectify the unrealistic shear strain energy component. The values of SCF are quite dispersed between each problem and may be difficult to determine [6]. To ensure smooth distribution of shear stress, various types of higher-order shear deformable theory (HSDT), which include higher-order terms in the approximation of the displacement field, were then devised. These higher-order shear deformation theories include, third-order shear deformation theory (TSDT) [7–11], fifth-order shear deformation theory (FiSDT) [12], trigonometric shear deformation theory [13–18], exponential shear deformation theory (ESDT) [19–21], refined plate theory (RPT) based on two unknown functions of transverse deflection [22] and others. It is worth mentioning that the HSDT models provide better results and yield more accurate and stable solutions (e.g. inter-laminar stresses and displacements) [12,18] than that of the FSDT without the SCF. However, the HSDT requires C^1-continuity of generalized displacement field leading to the second-order derivative of the stiffness formulation and thus causes difficulties in the standard finite element formulations. Several C^0 continuous elements [23–25] were proposed or alternatively Hermite interpolation function with the C^1-continuity was added for the specific approximation of transverse displacement [7]. These aforementioned methodologies may produce extra unknown variables including derivatives of deflection $w_{,x}$, $w_{,y}$, $w_{,xy}$ [26] which lead to significant computational costs. In recent years, C^1 continuous elements based on meshfree method [27–34] were proposed to solve the plate and shell problems. In this paper, we demonstrate that the C^1-continuous elements can be easily achieved by adopting isogeometric analysis without any additional variables.

Isogeometric analysis has been recently proposed by Hughes et al. [35] to bridge the gap between Computer Aided Design (CAD) and Finite Element Analysis (FEA). It refers to a computational framework in which the basis functions generated from non-uniform rational B-splines (NURBS) used to represent the geometry in CAD are used for approximating the unknown fields in FEA. Therefore, the process of meshing in IGA can be omitted and the two models for CAD and FEA can be integrated into one. The main advantages of IGA are the ability to represent domains with conic sections exactly and achieve higher order approximation with arbitrarily high smoothness. In IGA, the exact geometry is maintained at the coarsest level of discretization and re-meshing is performed on this level without any further communication with CAD geometry. Furthermore, B-splines (or NURBS) provide a flexible way to perform refinement (or *h*-refinement), and degree elevation [36]. Applications of IGA [37] can be found in several fields including structural mechanics, solid mechanics, fluid mechanics and contact mechanics. IGA has also been applied in analyzing the plate/shell structures such as, Kirchhoff–Love shells [38–40], isotropic Reissner–Mindlin shells [41], rotation-free shells [41], laminated composite/functionally graded plates based on the first order shear deformation theory [42–44], laminated composite/functionally graded plates based on the higher order shear deformation theory [45–47] and laminated composite plates based on the layer-wise theory [48].

In this paper, a higher order displacement fields in which the in-plane displacement expressed as inverse trigonometric functions of the thickness coordinate with constant transverse displacement across the thickness is presented. The present theory is combined with isogeometric analysis to create an efficient computational tool for analyzing the behavior of FGM isotropic and sandwich plates. The proposed inverse trigonometric functions can be expressed by means of the Taylor expansion, which has more general form than the classical polynomial. The displacement fields in the proposed theory are constructed using the NURBS basis functions that can yield higher-order continuity and therefore can easily fulfill the requirement of C^1-continuity of the transverse displacement passing through boundary of elements. This is an advantage over the finite element method. The material property varying continuously through plate thickness is homogenized by the rule of mixture or the Mori–Tanaka homogenization technique. Numerous numerical examples are presented in this paper to illustrate the effectiveness of the proposed formulation in comparison with other models from the literature.

This paper is organized as follows. Next section introduces the generalized HSDT for FGM plates. In Section 3, the formulation of HSDT based on IGA is described. The numerical results and discussions are provided in Section 4. Finally, in Section 5, concluding remarks are presented with the brief discussion on the numerical results obtained by the developed methodology.

2. The generalized higher order shear deformation theory for FGM plates

2.1. Problem definition

Consider a rectangular plate with length *a*, width *b* and a uniform thickness *h*, as shown in Fig. 1. Three different types of functionally graded material are studied in this paper: (1) isotropic FGM plates; (2) sandwich plates with FGM core and isotropic skins; (3) sandwich plates with isotropic core and FGM skins.

2.1.1. Isotropic FGM plates (type A)

Isotropic functionally graded material is a composite material created by mixing two distinct material phases which are often ceramic at one end and metal at the other. In this paper, two homogenous models namely the rule of mixture and the Mori–Tanaka technique have been used to estimate the effective properties of the FGM plate. The volume fraction of the ceramic and metal phase is assumed to vary in a continuous manner across thickness as described by the following function [7],

$$V_c(z) = \left(\frac{1}{2} + \frac{z}{h}\right)^n, \quad z \in [-h/2, h/2]; \quad V_m = 1 - V_c \tag{1}$$

where subscripts *m* and *c* refer to the metal and ceramic constituents, respectively. The Eq. (1) demonstrates that the volume fraction varies through the thickness depending on the power index *n*. Now, the effective properties of material according to the rule of mixture [7] are given by,

$$P_e = P_c V_c(z) + P_m V_m(z) \tag{2}$$

where P_c, P_m denote the individual material properties of the ceramic and the metal, respectively. The material properties that can be described by Eq. (2) include, Young's modulus (E), Poisson's ratio (ν), and density (ρ).

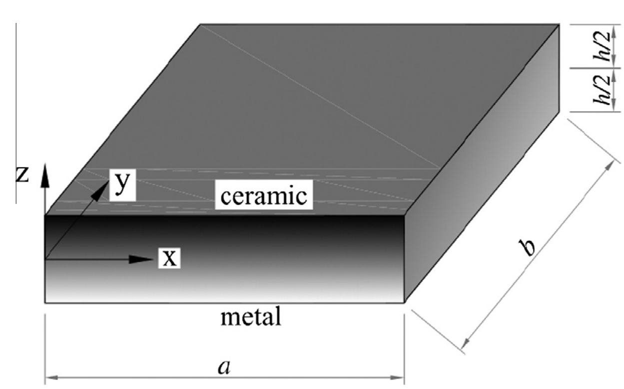

Fig. 1. A typical configuration of FGM based plate.