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Arbitrary active constrained layer damping treatments on beams: Finite element modelling and experimental validation

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Abstract

This paper concerns the analytical formulation and finite element modelling of arbitrary *active constrained layer damping* (ACLD) treatments applied to beams. A partial layerwise theory is utilized to define the displacement field of beams with an arbitrary number of elastic, viscoelastic and piezoelectric layers attached to both surfaces, and a fully coupled electro-mechanical theory is considered for modelling the behavior of the piezoelectric layers. The damping of the viscoelastic layers is modelled by the complex modulus approach. The weak forms of the analytical formulation, governing the motion and electric charge equilibrium, are presented. Based on the weak forms, a one-dimensional finite element (FE) model is developed, with the nodal mechanical degrees of freedom being the axial displacement, transverse displacement and the rotation of the mid-plane of the host beam and the rotations of the individual layers, and the electrical elemental degrees of freedom being the electrical potential difference of each piezoelectric layer. Frequency response functions were measured experimentally and evaluated numerically for a freely suspended aluminium beam with an ACLD patch. In order to validate the FE model the results are presented and discussed.

Keywords: Beam; Viscoelastic; Piezoelectric; Active constrained layer damping; Layerwise; Finite element

1. Introduction

Passive damping treatments have been extensively used in engineering to reduce vibration and noise radiation [1–3]. The simplest form of passive damping is the one where single layers of viscoelastic materials are attached to the host structure. This is known as *passive layer damping* (PLD). When the structure vibrates, energy is dissipated in the viscoelastic layer. Increasing the thickness and length of the viscoelastic treatment would increase the energy dissipation and consequently the damping. However, in applications where the weight is of critical importance, a more efficient treatment is required, and other alternatives to increase damping must be found.

It is well known that the inclusion of elastic constraining layers covering the viscoelastic layers can enhance the energy dissipation through an increase in shear deformations. This type of treatment is known as *passive constrained layer damping* (PCLD). Some examples can be found in Refs. [4–6]. However, while passive damping treatments can greatly improve the damping of the system, there are some limitations. Viscoelastic materials have frequency and temperature dependent mechanical properties which can make the damping change, bringing limitations to the effective temperature and frequency

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range of the treatment. In order to provide adequate damping over a broad frequency band, different viscoelastic materials must be chosen which often complicates the analysis and the design of the system. Therefore, while viscoelastic treatments are easy to apply, the damping is often of limited bandwidth.

From the early 1990s so-called *active constrained layer damping* (ACLD) treatments have been analysed and applied to structures [7–9]. These are hybrid treatments with constraining layers made of piezoelectric materials. One of the unique features of piezoelectric materials is that they can be used both as sensors and actuators [10]. If utilized as actuators, and according to an appropriate control law, the active constraining layer can increase the shear deformation of the viscoelastic layer and overcome some of the PCLD limitations. The ACLD treatments combine the high capacity of passive viscoelastic materials to dissipate vibrational energy at high frequencies with the active capacity of piezoelectric materials at low frequencies. Therefore, in the same damping treatment, a broader band control is achieved benefiting from the advantages of both passive (simplicity, stability, fail-safe, low-cost) and active (adaptability, high-performance) systems. Some examples can be found in the works of Baz and his co-workers [11–13].

Various configurations of active and passive layers have been proposed in an attempt to improve performance. In general the so-called hybrid active–passive (or arbitrary ACLD) treatments involving arbitrary arrangements of constraining and passive layers, integrating piezoelectric sensors and actuators, may be utilized [14–21]. A survey of advances in hybrid active–passive vibrations and noise control via piezoelectric and viscoelastic constrained layer treatments can be found in Refs. [22–24].

Modeling this kind of structural system often requires a coupled model of the structure, which comprises piezoelectric, viscoelastic and elastic layers. These treatments are applied to beams, plates and shells. They can be modelled as either lumped or distributed parameter systems, and usually have complicated geometries that make analytical solution of the equations of motion difficult, if not impossible. Alternatively, various modelling techniques, such as finite element (FE) modelling, modal analysis, and lumped parameters models, allow the approximation of the partial differential equations by a finite set of ordinary differential equations.

The temperature and frequency dependent properties of the viscoelastic materials causes some difficulties for the mathematical model, increasing its complexity. Usually the temperature is assumed constant and only models concerning frequency dependence are utilized. The simplest way of modelling those materials is achieved by a *complex modulus approach* (CMA) where the material properties are defined for each frequency value. The CMA is a frequency domain method that is limited to steady state vibrations and single-frequency harmonic excitations [25,26]. Time domain models such as the *Golla–Hughes–McTavish* (GHM) [27,28], *anelastic displacement fields* (ADF) [29,30] or *fractional calculus* [31,32] models, have been developed in the last few years and represent good alternatives to the CMA allowing a reduction in the computational effort and the study of the transient response in a more straightforward manner.

In the development of FE models with piezoelectric actuators or sensors, different assumptions can be taken into account in the theoretical model when considering the electro-mechanical coupling. A survey on the advances in FE modelling of piezoelectric adaptive structures is presented by Benjeddou [33]. These assumptions regard mainly the use (or not) of electric degrees of freedom (DoFs) and the approximations of the through-the-thickness variation of the electric potential. Therefore, they lead to decoupled, partial and fully coupled electro-mechanical theories, which in turn can lead to different modifications of the structure's stiffness and different approximations of the physics of the system. Those electro-mechanical coupling theories can be considered by the use of *effective stiffness parameters*, defined according to the electric boundary condition considered, as shown in [34,35] for a smart piezoelectric beam.

When designing hybrid active–passive treatments it is important to know the configuration of the structure and treatment that gives optimal damping. For simulation the designer needs a model of the system in order to define the optimal locations, thicknesses, configurations, control law, etc. Thus, there are numerous options at the design stage.

In this paper, a generic analytical formulation that accounts for the hybrid couplings of active–passive treatments on beams is developed and a structural analytical model of a composite beam with an arbitrary number of layers of elastic, piezoelectric and viscoelastic materials, attached to both surfaces of the beam, is derived. The kinematic assumptions, based on a partial layerwise theory, are first presented. Then, the constitutive equations and the electrical model assumptions for the piezoelectric materials, which account for a fully coupled electro-mechanical theory, are described. Moreover, the frequency and temperature dependent constitutive behavior of the viscoelastic layers is discussed and the damping behavior of the viscoelastic layers is modelled by the CMA. Hamilton's principle is utilized to derive the weak forms governing the motion and electric charge equilibrium of the beam with arbitrary ACLD treatments. Based on the weak forms a FE solution is presented and a composite beam FE is developed. Finally, a case study concerning a freely suspended aluminium beam with a partial ACLD treatment (viscoelastic layer sandwiched between the beam and piezoelectric patch) is analyzed. The developed FE is used in the prediction of three frequency response functions: acceleration per unit force, acceleration per unit voltage into the piezoelectric actuator and induced voltage per unit force. The predicted results are presented and compared with the measured ones in order to validate the FE model.

There are several published works in the open literature that consider in their models mechanical and electrical assumptions similar to the ones presented here (e.g., [18,36]). However, they are generally limited to models of three-layered