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Detection of damage in cyclic structures using an eigenpair sensitivity matrix

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ABSTRACT

In cyclic structures, the eigenvectors associated with repeated eigenvalues do not change smoothly in case of damages, thus, the existing sensitivity-based damage detection techniques are rendered ineffective. To overcome this problem, a transformation from such eigenvectors to a set of well-behaved pseudo-eigenvectors is introduced. The proposed pseudo-eigenvectors can be linearly approximated, and hence, they admit sensitivity-based analysis. Subsequently, sensitivity matrices of eigenvalues and pseudo-eigenvectors are established, and four damage detection algorithms are developed accordingly. Simulation results under different number of sensors and various noise levels confirm the efficiency of the proposed approach.

1. Introduction

Structural damage detection can be viewed as solving a nonlinear system of equations in which the damage variables should be determined so that the analytical responses of the structure match the measured ones in an optimal way [1]. The mathematical expression of the problem is:

$$\mathbf{R}_d = \mathbf{R}(\mathbf{X}) \Rightarrow \mathbf{X} = ? \tag{1}$$

where $\mathbf{X} = (x_1, x_2, \ldots, x_n)^T$ is the damage vector, in which $0 \leqslant x_i \leqslant 1$ is the damage ratio of the ith element ($x_i = 0$ and $x_i = 1$ indicate the intact and completely damaged states, respectively), n is the number of structural elements, $\mathbf{R}_d = (r_{d,1}, r_{d,2}, \ldots, r_{d,m})^T$ is the vector of m structural responses of the existing damaged structure and $\mathbf{R}(\mathbf{X}) = (r_1(\mathbf{X}), r_2(\mathbf{X}), \ldots, r_m(\mathbf{X}))^T$ is the vector of m responses of a hypothetically damaged structure that can be evaluated by the analytical model.

If the number of unknowns is less than the number of equations, the corresponding system of equations will generally be over-determined; therefore, an error between two sides of Eq. (1) occurs that should be minimized. On the flip side, if the number of unknowns is more than the number of equations, the corresponding system of equations will be under-determined which has infinite solutions. It should be noted that in the damaged structure most of the structural elements are still intact, thus the right solution has high sparsity [1,2]. This fact can help one to find the actual damage solution in under-determined system of equations. For under-determined linear system of equations under certain conditions, finding the sparsest solution (a solution with minimum ℓ_0 norm) is equivalent to finding the solution with minimum ℓ_1 norm [3–5]. A solution with minimum ℓ_1 norm can be easily found by linear programming; hence it is highly preferred rather than that minimization of ℓ_0 which is mostly an NP-hard problem. This fact is well considered for compressed sensing in signal processing [3–5]. Meruane and Heylen [6] adopted the above concept for damage detection through adding the summation of damage variables as a penalty function to the conventional objective function.

An acceptable approximation is obtained by linearizing the problem about the intact state and solving it by Moore–Penrose pseudo-inverse. In previous work, damage detection through once or iteratively applying linear approximations has been proceeded by various responses [7–9]. Lu and Law [8] established sensitivity matrix for the forced vibration response in time domain which is afterwards modified by regularization. Then, they detected the damages by solving the linearized equation. Koh and Dyke [9] used correlation of frequencies of the real damaged and hypothetically damaged structures, namely multiple damage location assurance criterion (MDLAC) which has been first introduced by Messina et al. [10], as the objective function to be maximized through genetic algorithm. Au et al. [11] by considering the differences of elemental modal strain energy between undamaged and damaged states, first restricted the potentially damaged elements and then, searched for the damage scenario in the reduced search space by applying micro genetic algorithm (μ-GA). Lallement and Prianda [12] introduced a robust subset selection scheme abbreviated forward selection. In each step of forward selection, the parameter with the most contribution to the response change vector is selected as the damage parameter. Friswell and coworkers employed

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this method frequently and confirmed its effectiveness [13–15]. To select the number of damage parameters in forward selection, Friswell et al. employed Efroymson's criterion in Ref. [14]. In that work, forward selection is extended to the case with multiple measurement sets using angles between subspaces. Friswell et al. [15] also extended forward selection for parameter groups. They employed Tikhonov regularization for stabilizing the method. Fritzen et al. [16] presented a strategy similar to forward selection scheme by using QR decomposition. In Ref. [17] Friswell gave a comprehensive overview of the regularization methods through subset selection. Weber et al. [18] considered the eigenvalues and eigenvectors as the structural response to solve the corresponding linearized equation. They regarded that the sensitivity of the structural response with respect to the elemental damage was badly conditioned. Consequently, both sides of the linearized equation were multiplied by weight matrix and the coefficient matrix of the equation and then modified by regularization using truncated singular value decomposition and Tikhonov methods. As in practice the noise level is unknown, in a similar work, Chen [19] employed the generalized cross validation to optimize the truncation parameter of a method of regularization called truncated singular value decomposition. Perera and Ruiz [20] revealed that the ratio of the change in mass normalized eigenvectors to the corresponding eigenvalues change is lowly sensitive to the modeling errors, and based on that they presented two objective functions for their optimization process of the damage detection. Gui and Li [21] used the frequency changes and the mode shape changes as two different information sources for considering in fusion center to localize damages using evidence theory. Then, in the second stage, a micro-search genetic algorithm was proposed to determine the damage extents. While the cracks were modeled in the corresponding joints of the structure by using a hinge and a rotational spring, Lee [22] applied Newton–Raphson method to identify the multiple cracks of the beams using natural frequencies. In our previous work [1], the sensitivity analysis as an operator was added to the genetic algorithm body. In that novel operator, each individual was improved by solving the linearized problem using pseudo-inverse. Another new operator for restricting the elements through optimization was embedded in genetic algorithm body. Numerical results demonstrated the high efficiency of the proposed method compared to those found in the literature.

Cyclic structures are rotationally periodic structures. In cyclic structures, some or all of the eigenvalues are repeated twice, while in the other structures repetition of eigenvalues is a rare phenomenon. The structural mode shapes associated with repeated eigenvalues are very unstable and behave highly nonlinear and non-monotonic with respect to change of the design parameters [23]. Structures with repeated eigenvalues and their modal derivatives are well-recognized and have been widely investigated in the literature [23–29]. However, as the best of our knowledge, damage detection of such structures using sensitivity analysis of eigenpairs has not been studied yet.

In the present work, the derivatives of eigenpairs of the cyclic structures are mathematically declared, and a modified response associated with repeated eigenvalues with continuity and linearity properties which can be used instead of the related eigenvectors, is introduced. Also, several algorithms for estimating structural damages through just one time establishing the sensitivity matrix are presented. Moreover, effectiveness of the proposed algorithms for various sensor positions and noise levels is compared.

The present paper is organized as follows: Sensitivities of eigenvalues and eigenvectors for the repeated and distinct eigenvalues are described in Section 2. In Section 3, pseudo-eigenvector as a modified response is introduced. The damage detection algorithms are presented in Section 4. Three case studies are considered in Section 5. Conclusions are derived in Section 6. An appendix is given at the end.

2. Sensitivities of eigenpairs

This paper proposes damage detection by considering eigenvalue and eigenvector changes, thus computation of eigenpair derivatives is needed. In the following, the derivative of eigenvalues and eigenvectors with respect to a design variable for the cases of distinct and repeated eigenvalues are presented.

2.1. Distinct eigenvalues

The derivative of the ith structural eigenvalue, λ_i, which is distinct, with respect to a design variable, say b, can be obtained using the following formula [30]:

$$\frac{\partial \lambda_i}{\partial b} = \boldsymbol{\phi}_i^T \frac{\partial \mathbf{K}}{\partial b} \boldsymbol{\phi}_i - \lambda_i \boldsymbol{\phi}_i^T \frac{\partial \mathbf{M}}{\partial b} \boldsymbol{\phi}_i \tag{2}$$

where $\mathbf{K}$ and $\mathbf{M}$ are the structural stiffness and mass matrices and $\boldsymbol{\phi}_i$ is the ith eigenvector. The derivative of the ith eigenvector with respect to the design variable can be evaluated using the following relationship [30]:

$$\frac{\partial \boldsymbol{\phi}_i}{\partial b} = \sum_{j=1}^{n} c_{ij} \boldsymbol{\phi}_j \tag{3}$$

in which the c_{ij} can be evaluated for the cases of $i = j$ and $i \neq j$ as follows:

$$c_{ii} = \frac{1}{2} \boldsymbol{\phi}_i^T \frac{\partial \mathbf{M}}{\partial b} \boldsymbol{\phi}_i \quad \text{for } i = j \tag{4 - a}$$

$$c_{ij} = \frac{\boldsymbol{\phi}_j^T \left(\frac{\partial \mathbf{K}}{\partial b} - \lambda_i \frac{\partial \mathbf{M}}{\partial b} \right) \boldsymbol{\phi}_i}{\lambda_i - \lambda_j} \quad \text{for } i \neq j \tag{4-b}$$

2.2. Repeated eigenvalues

Let us consider a structure with a repeated eigenvalue with multiplicity of q as $\lambda_k = \lambda_{k+1} = \ldots = \lambda_{k+q-1}$ and the related mass normalized eigenvectors in matrix form as $\boldsymbol{\Phi} = [\boldsymbol{\phi}_k, \boldsymbol{\phi}_{k+1}, \ldots, \boldsymbol{\phi}_{k+q-1}]$. Hence, the corresponding eigenvalue problem is $\mathbf{K}_{p\times p}\boldsymbol{\Phi}_{p\times q} = \mathbf{M}_{p\times p}\boldsymbol{\Phi}_{p\times q}\boldsymbol{\Lambda}_{q\times q}$, where $\boldsymbol{\Lambda} = \text{diag}\,(\lambda_i; i = k, \ldots, k + q - 1) = \lambda \mathbf{I}_{q\times q}$.

Because of the repetition of eigenvalues, the associated eigenvectors are not unique, and they can also be considered as any other bases of the eigenspace span($\boldsymbol{\Phi}$). Therefore, in general $\boldsymbol{\Psi} = \boldsymbol{\Phi}_{p\times q}\mathbf{H}_{q\times q}$ is another set of associated eigenvectors where $\mathbf{H}$ can be any matrix having the property of $\mathbf{H}^T\mathbf{H} = \mathbf{I}$ for mass orthonormality condition. However, while the system is perturbed due to damage, the repeated eigenvalue splits into distinct eigenvalues and as a result, the corresponding eigenspace is decomposed into several distinct eigenvectors. To determine the new eigenvectors, one has to properly find the unique $\mathbf{H}$. The matrix $\mathbf{H}$ and also the derivatives of eigenvalues with respect to the design variable can be determined using the following eigenvalue problem [25–28]:

$$\mathbf{QH} = \mathbf{H} \frac{\partial \boldsymbol{\Lambda}}{\partial b} \tag{5}$$

where $\mathbf{Q} = \boldsymbol{\Phi}^T \left[\frac{\partial \mathbf{K}}{\partial b} - \lambda \frac{\partial \mathbf{M}}{\partial b} \right] \boldsymbol{\Phi}$. The derivative of the set of eigenvectors $\boldsymbol{\Psi}$ with respect to design variable b can be obtained as follows:

$$\frac{\partial \boldsymbol{\Psi}}{\partial b} = \mathbf{W} + \boldsymbol{\Psi}\mathbf{C} \tag{6}$$

where the diagonal and off-diagonal entries of matrix $\mathbf{C}$ are defined as follows: