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Improving the MITC3 shell finite element by using the Hellinger–Reissner principle

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ABSTRACT

The objective of this study is to improve the performance of the MITC3 shell finite element. The Hellinger–Reissner (HR) variational principle is modified in the framework of the MITC method, and a special approximated transverse shear strain field is proposed. The MITC3-HR shell finite element improved by using the Hellinger–Reissner functional passes all the basic tests (zero energy mode test, patch test, and isotropic element test). Convergence studies considering a fully clamped plate problem, a sixty-degree skew plate problem, cylindrical shell problems, and hyperboloid shell problems demonstrate the improved predictive capability of the new 3-node shell finite element.

1. Introduction

Shell structures have been widely used in many engineering applications, and the finite element method has been dominantly adopted for the analysis of shells. Since the effectiveness of a shell finite element analysis depends highly on the predictive capability of shell finite elements, there is strong demand for the development of more effective shell finite elements [1–3].

The kinematical behavior of shells is very complicated, especially as the shell thickness becomes smaller. Depending on shell geometries, boundary conditions, and applied loadings, the asymptotic behavior can be dominated by membrane or bending actions or a combination of both actions [3–10]. A reliable shell finite element should provide accurate solutions irrespective of the complicated asymptotic behaviors and the magnitude of the shell thickness. However, it is extremely challenging to develop such shell finite elements due to a locking phenomenon; that is, the shell finite element becomes too stiff when the thickness is relatively small in bending situations [1,11].

The MITC (Mixed Interpolation of Tensorial Components) method has been successfully used to develop shell finite elements to reduce the locking effects [12–17]. The MITC4 quadrilateral shell finite elements were first developed by Dvorkin and Bathe [12,13]. The MITC9 and MITC16 quadrilateral shell finite elements were later developed by Bucalem and Bathe [14]. In spite of the fact that triangular elements are very useful for the mesh generation of arbitrary shell geometries, due to the relatively complicated representation of triangular geometries in the element formulation, the MITC method was recently applied to develop isotropic triangular shell elements, MITC3 and MITC6, by Lee and Bathe [15].

The MITC3 triangular shell finite element shows much better predictive capability than the displacement-based 3-node triangular shell finite elements and other 3-node isotropic triangular shell finite elements [15,18]. However, the locking alleviation by MITC3 is not as large as that by MITC4; that is, the accuracy of the solutions is not as good as that of the MITC4 quadrilateral shell finite elements due to locking. This provided the motivation for this work.

The purpose of this paper is to improve the MITC3 shell finite element. With the help of the Hellinger–Reissner (HR) principle [1], we additionally approximate the transverse shear strain fields of the MITC3 shell finite element. The Hellinger–Reissner (HR) functional has been used to alleviate locking in plate and shell finite elements [19,20]. The successful use of this method depends on how the approximated transverse shear strain fields are constructed. We first modify the Hellinger–Reissner functional and introduce a special approximated transverse shear strain field based on rotated contravariant bases.

In the following sections, the MITC3 formulation is briefly reviewed and the Hellinger–Reissner principle for the shell finite element is presented. We then propose a method that involves the use of the Hellinger–Reissner functional to improve the MITC3 shell finite element, after which we explain how to construct the special approximated transverse shear strain field. The basic test results and well-established convergence studies numerically show that the MITC3 shell finite element is successfully improved.

2. Formulation of the MITC3 shell finite element

The geometry of a 3-node continuum mechanics based triangular shell finite element is interpolated by Lee and Bathe [15] and Lee et al. [18]

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$$\vec{x}(r,s,t)=\sum_{i=1}^{3}h_i(r,s)\vec{x}_i+\frac{t}{2}\sum_{i=1}^{3}a_i h_i(r,s)\overrightarrow{V}_n^i, \tag{1}$$

where $h_i(r,s)$ is the 2D shape function of the standard isoparametric procedure corresponding to node i, $\vec{x}_i$ is the position vector for node i in the global Cartesian coordinate system, and a_i and $\overrightarrow{V}_n^i$ denote the shell thickness and the director vector at node i, respectively; see Fig. 1.

The displacement interpolation of the element is obtained by

$$\vec{u}(r,s,t)=\sum_{i=1}^{3}h_i(r,s)\vec{u}_i+\frac{t}{2}\sum_{i=1}^{3}a_i h_i(r,s)(-\overrightarrow{V}_2^i\alpha_i+\overrightarrow{V}_1^i\beta_i), \tag{2}$$

in which $\vec{u}_i$ is the nodal displacement vector in the global Cartesian coordinate system, $\overrightarrow{V}_1^i$ and $\overrightarrow{V}_2^i$ are unit vectors orthogonal to $\overrightarrow{V}_n^i$ and to each other, and α_i and β_i are the rotations of the director vector $\overrightarrow{V}_n^i$ about $\overrightarrow{V}_1^i$ and $\overrightarrow{V}_2^i$ at node i.

The linear terms of the displacement-based covariant strain components are given by

$$e_{ij}=\frac{1}{2}(\vec{g}_i\cdot\vec{u}_{,j}+\vec{g}_j\cdot\vec{u}_{,i}), \tag{3}$$

where

$$\vec{g}_i=\frac{\partial\vec{x}}{\partial r_i},\quad \vec{u}_{,i}=\frac{\partial\vec{u}}{\partial r_i}\qquad \text{with } r_1=r,\ r_2=s,\ r_3=t. \tag{4}$$

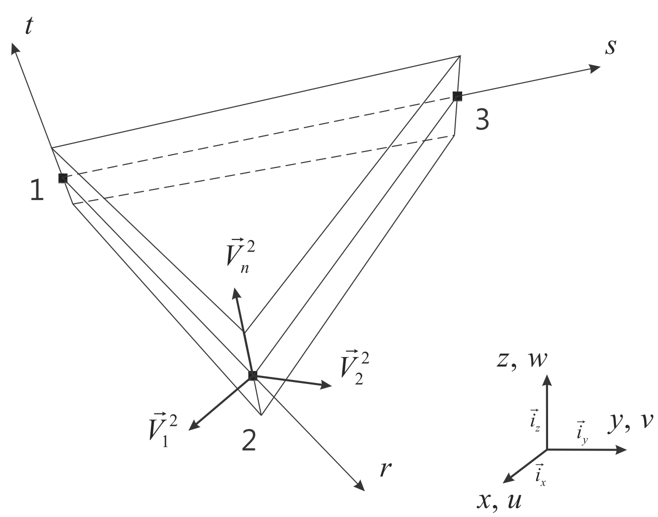

Fig. 1. A 3-node triangular continuum mechanics based shell finite element.

The covariant strain components can be expressed by

$$e_{ij}=\boldsymbol{b}_{ij}\overrightarrow{U}, \tag{5}$$

in which $\boldsymbol{b}_{ij}$ is the covariant strain–displacement matrix and $\overrightarrow{U}$ is the vector of nodal displacements and rotations, which include $\vec{u}_k$, α_k, and β_k.

The base vectors of the shell-aligned local Cartesian coordinate system are defined as follows

$$\overrightarrow{L}_{\bar{t}}=\frac{\vec{g}_t}{|\vec{g}_t|},\quad \overrightarrow{L}_{\bar{r}}=\frac{\vec{g}_s\times\overrightarrow{L}_{\bar{t}}}{|\vec{g}_s\times\overrightarrow{L}_{\bar{t}}|},\quad \overrightarrow{L}_{\bar{s}}=\overrightarrow{L}_{\bar{t}}\times\overrightarrow{L}_{\bar{r}}. \tag{6}$$

The strains (ε_{ij}) defined in the shell-aligned local Cartesian coordinate are calculated from the covariant strain components through the following relation,

$$\varepsilon_{ij}(\overrightarrow{L}_i\otimes\overrightarrow{L}_j)=e_{mn}(\vec{g}^m\otimes\vec{g}^n)\quad \text{with } \overrightarrow{L}_1=\overrightarrow{L}_{\bar{r}},\ \overrightarrow{L}_2=\overrightarrow{L}_{\bar{s}},\ \overrightarrow{L}_3=\overrightarrow{L}_{\bar{t}}. \tag{7}$$

In Eq. (7), the contravariant base vectors $\vec{g}^j$ are given by

$$\vec{g}_i\cdot\vec{g}^j=\delta_i^j\quad \text{with } \vec{g}^1=\vec{g}^r,\ \vec{g}^2=\vec{g}^s,\ \vec{g}^3=\vec{g}^t, \tag{8}$$

where δ_i^j is the Kronecker delta in mixed form.

The strain vector defined in the shell-aligned local Cartesian coordinate system from the displacement-based shell formulation is

$$\vec{\varepsilon}=\boldsymbol{B}\overrightarrow{U}, \tag{9}$$

where $\vec{\varepsilon}=[\varepsilon_{\bar{r}\bar{r}}\quad \varepsilon_{\bar{s}\bar{s}}\quad 2\varepsilon_{\bar{r}\bar{s}}\quad 2\varepsilon_{\bar{s}\bar{t}}\quad 2\varepsilon_{\bar{r}\bar{t}}]^T$.

In the formulation of the MITC3 shell finite element, the covariant in-plane strain field is calculated by the displacement-based triangular shell formulation in Eq. (3) and the MITC method is only applied to substitute the covariant transverse shear strain field [15]. The assumed covariant transverse shear strain components, which are spatially isotropic, are given by

$$\tilde{e}_{rt}=e_{rt}^{(1)}+cs,\quad \tilde{e}_{st}=e_{st}^{(2)}-cr, \tag{10}$$

where $c=e_{st}^{(2)}-e_{rt}^{(1)}-e_{st}^{(3)}+e_{rt}^{(3)}$ and, at the tying points, $e_{rt}^{(n)}$ and $e_{st}^{(n)}$ are calculated from Eq. (3), see Fig. 2.

The assumed covariant transverse shear strain components of the MITC3 element can also be expressed by

$$\tilde{e}_{ij}=\tilde{\boldsymbol{b}}_{ij}\overrightarrow{U}. \tag{11}$$

The covariant strains of the MITC3 shell finite element are transformed to the strains defined in the shell-aligned local Cartesian coordinate system $(\overrightarrow{L}_{\bar{r}},\overrightarrow{L}_{\bar{s}},\overrightarrow{L}_{\bar{t}})$

$$\vec{\varepsilon}^M=\boldsymbol{B}^M\overrightarrow{U}\quad \text{with } \vec{\varepsilon}^M=[\varepsilon_{\bar{r}\bar{r}}^M\quad \varepsilon_{\bar{s}\bar{s}}^M\quad 2\varepsilon_{\bar{r}\bar{s}}^M\quad 2\varepsilon_{\bar{s}\bar{t}}^M\quad 2\varepsilon_{\bar{r}\bar{t}}^M]^T. \tag{12}$$

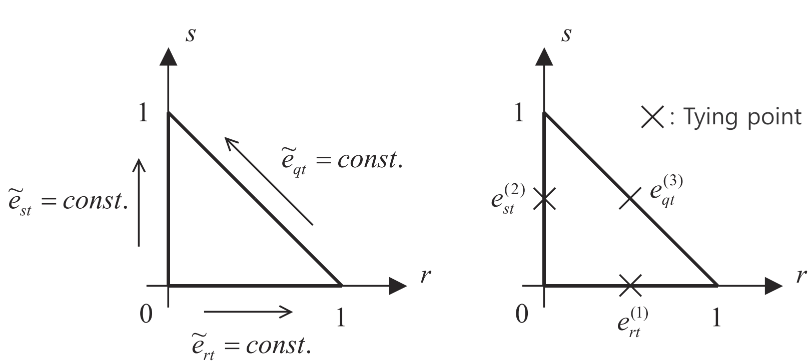

Fig. 2. Tying positions for the transverse shear strain of the MITC3 triangular shell finite element. The constant transverse shear strain condition is imposed along its edges.