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Design and analysis of a new filter for LES and DES

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ABSTRACT

Spatial filters have been rigorously analyzed here for stabilization and dispersion relation preservation properties using spectral-matrix theory.

Traditional LES faces problems of instability, aliasing and additional complexity of SGS modeling. Here, an alternative is proposed where unfiltered governing equation is solved, followed by spatial upwind filtering, without requiring any SGS models. This filter retains the resolution of high accuracy methods and removes energy, like the action of hyper-viscosity used in spectral methods. As examples, solutions for (a) accelerated flow past a NACA-0015 airfoil held normal to the flow and (b) transitional flow past a NLF airfoil, have been reported.

1. Introduction

Filtering is always implicitly present in all discrete computations – as explained in [1–3] using Fourier–Laplace transform technique. At the outset, we emphasize that the performance of a filter is judged by its ability to leave smaller wave number components of a variable unaffected while filtering the higher wave number components – those known to create numerical problems. It is easy to show the equivalence of discrete operations with low-pass filtering of unknowns. In contrast, in traditional LES the governing equation is analytically filtered before discretization. See e.g. in [4–10] for the effects of different types of filters used in traditional LES. Explicit Padè filters have been proposed in [11–14] to control instabilities arising from mesh non-uniformities and the application of numerical boundary conditions. The same authors now advance the use of spatial filters as a tool for LES [15,16]. These spatial filters are central in nature for the interior nodes, while one-sided boundary filters have been proposed for non-periodic problems [12,17]. Present analysis shows that these boundary filters can be destabilizing near the inflow of a computational domain, as shown here by a global analysis for the first time using matrix- spectral theory. This instability affects more number of points, with increase in the order of interior filters. Also, the filtering operation in traditional LES requires adding additional stresses (termed as SGS or Leonard stresses) at the formulation stage itself. In contrast, the spatial filters [11–16] are applied at the end of time advancement of governing equations, without the need for any stress modeling. Although, no strict rules have been proposed so far for the frequency of filtering. Thus, there is a significant advantage on the accuracy of LES performed using spatial filters for various reasons. Firstly, performing LES with spatial filters, one can completely dispense with SGS model – an empirical process that is mandatory in traditional LES. Secondly, this approach involves less computation, as one needs to perform spatial filtering in the physical space. Most importantly, this can also help avoiding numerical instabilities due to aliasing operation involved in traditional LES. Note that in LES using explicit spatial filter [15–17] or implicit filter in higher order upwinding methods [18] one only band-limits the variable through the filtering operation. In contrast, traditional LES filters the governing equation at the formulation stage by convoluting various terms with another space-dependent filtering function. This process can lead to aliasing and is explained next with the help of the following linear convection equation:

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0 \tag{1}$$

Solving this equation by an upwind method is equivalent to attenuating the unknown u, with high wave number components removed by the implicit filter that results in severe loss of signal [19]. Solving the same equation by a non-dissipative scheme and then band-limiting by spatial filtering does not lead to this loss of signal. In contrast, explicit filtering in traditional LES converts Eq. (1) to,

$$\frac{\partial \bar{u}}{\partial t} + c\frac{\partial \bar{u}}{\partial x} = 0$$

where $\bar{u} = \int F(x - x')u(x')dx'$.

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The filtering operation here is equivalent to taking product of two space-dependent functions that can lead to aliasing – even for a linear equation! Detailed discussion on aliasing for physical plane computations can be found in [2]. Actual aliasing depends upon the band-width of the filter function, F and the function representing the unknown and its derivatives. While this is a possibility for a linear equation, the prospect of aliasing for nonlinear equation is even more severe. Thus, for the solution of Navier–Stokes equation, such convolution can produce just the opposite of desired effect due to aliasing [2].

Filtering associated with discretization is equivalent to low-pass filtering via the real part of the transfer function, while any present negative imaginary part, attenuates the signal further by the added numerical dissipation. It is well-known that the implicit filtering of first derivative is stronger than that for the second derivatives, at high wave numbers near the Nyquist limit [2]. Thus, for solving the Navier–Stokes equation, the loss of accuracy due to discretization becomes prominent, for the convective terms rather than the diffusion terms. In traditional LES, filtering the governing equation, creates an energy sink via the SGS model that interrupts existing energy cascade. In numerical methods using higher order upwinding to discretize convection terms, implicitly added numerical diffusion plays a similar role across all wave numbers, specially more at higher wave numbers. In [19], this connection between LES and higher order upwinding was described.

While the above description was for the discretization process by symmetric and non-symmetric stencils, a similar interpretation holds for filtering operation also. While there might be similarity between the real and imaginary parts of transfer functions of the filters with the low-pass feature of filters and SGS models, the distinction between the two has to be explained clearly. An analytical attempt is made here to show the differences and similarities between the two. This confusion has prompted the authors in [16] to state that *due to the spectral-like dissipation properties of the filter, it also serves the same function as that of an SGS model without additional computational expense.* Present analysis follows the global spectral-matrix approach advanced in [19–21] described briefly below.

If an unknown is represented by its Fourier–Laplace transform [22,23] at the jth node, in a uniformly spaced grid of spacing h by, $u(x_j,t) = \int U(k,t)e^{ikx_j}dk$, then the exact derivative at the same node is given by,

$$[u_j']_{exact} = \left(\frac{\partial u}{\partial x}\right)_{exact} = \int ikUe^{ikx_j}dk \tag{2}$$

This is a global approach in characterizing of discretization, with the phase still determined by the point in question (x_j) only. In solving periodic problems, there is no distinction between local and global approaches. However, a global approach is needed for non-periodic problems. This issue of analysing discretization methods was proposed in [20,21] using Fourier–Laplace transform by using spectral-matrix analysis framework. In this method, one obtains k_{eq} for all the nodes of the computational domain simultaneously. Additionally, the numerical properties of stability, DRP properties and error analysis for any discrete method was obtained – as reported in [19,21].

In a finite-domain, the first derivative is numerically estimated for discrete computations (by any method) from, $[A_1]\{u'\} = \frac{1}{h}[B_1]\{u\}$. For explicit methods, $[A_1] \equiv [I]$, where $[I]$ is the identity matrix. This general form can also be written as $\{u'\} = \frac{1}{h}[C]\{u\}$, where $[C] = [A_1]^{-1}[B_1]$. Appropriate $[C]$-matrices for finite-domain non-periodic problems are provided for a range of explicit and implicit finite difference methods in [21] and for other generic methods are to be found in [2]. Here onwards, we will confine our discussion to finite difference methods only. The dimension of [C]-matrix is determined by the number of nodes. The top and bottom rows of $[C]$ correspond to boundary and near-boundary stencils for non-periodic problems. The derivative at the jth node is evaluated as $u_j' = \frac{1}{h}\sum_{l=1}^{N} C_{jl}u_l$, where $u_l = u(x_l,t) = \int U(k,t)e^{ikx_l}dk$ is the unknown at the lth node and N is the total number of nodes. In confirmity with spectral representation in Eq. (2), this numerical derivative is written as,

$$u_j' = \int \frac{1}{h}\sum_{l=1}^{N} C_{jl}U(k,t)e^{ik(x_l-x_j)}e^{ikx_j}dk \tag{3}$$

As the phase part of the representation in (2) and (3) are identical, one can write the following by comparing the two expressions:

$$i[k_{eq}]_j = \frac{1}{h}\sum_{l=1}^{N} C_{jl}e^{ik(x_l-x_j)} \tag{4}$$

Although the entries of $[C]$ are real, $[k_{eq}]_j$ is in general complex, with real part representing the implicit low-pass filter of the discretization. If the stencil for differentiation is non-symmetric, then one gets an imaginary part of k_{eq} that represents numerical dissipation or anti-diffusion, which either attenuates or amplifies the unknown, respectively [19,21]. These effects of the real and imaginary parts of k_{eq} become more pronounced at high wave numbers. Thus, both the real and imaginary parts of k_{eq} can help to band-limit the unknowns.

In [11,14], the problems of numerical instability caused by spatial discretization at high wave numbers were proposed to be removed by using spatial filters. Different order central filters were proposed along with their transfer functions given in [11,17]. Furthermore, boundary filters for non-periodic problems were also proposed in [12–14] and their transfer functions were obtained by a local method. The effect of filters on numerical instability was similarly obtained in isolation, for non-periodic problems. The application of local analysis technique to investigate boundary/near-boundary filters is inconsistent and is proposed to be replaced here by the full-domain approach of [19–21].

Apart from reporting transfer functions of filters and their effects on numerical amplification factors by a local analysis, there are no results for the full-domain non-periodic filters in the literature. One of the major reasons for undertaking the present research is to perform a rigorous analysis for transfer functions and numerical instability. More importantly, we report the DRP properties of filters for performing LES and DES.

The present paper is formatted in the following manner. In the next section, we describe the properties of the transfer function for both periodic and non-periodic filters. In Section 3, the numerical amplification factor and the DRP properties of various filters are provided. In Section 4, we propose a new upwind filter. Properties of the new upwind filter obtained by the present analysis, are demonstrated by solving two examples from fluid mechanics that involves resolving high wavenumber phenomenon in Section 5. We conclude the paper with a summary and conclusion in Section 6.

2. Transfer function of periodic and non-periodic filters

The structure of central Padè filters [11–13] makes use of easy solution of tridiagonal matrix equation for the filtered quantities denoted with caret, evaluated in terms of the unfiltered variables u on the right hand side of the equation given by,

$$\alpha\hat{u}_{j-1} + \hat{u}_j + \alpha\hat{u}_{j+1} = \sum_{n=0}^{M}\frac{a_n}{2}(u_{j+n} + u_{j-n}) \tag{5}$$